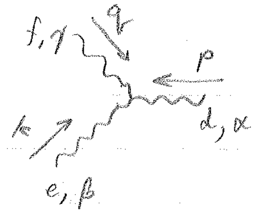
Feynman rule for $\quad + g f^{abc} \partial_\mu A^a_\nu A^{\mu b} A^{\nu c}$



$$\sim \quad \langle 0 | \underbrace{\left(g f^{abc} \partial_\mu A^a_\nu A^{\mu b} A^{\nu c} \right)}_{\text{Write as } g f^{abc} g^{\mu\rho} g^{\nu\sigma} \partial_\mu A^a_\nu A^b_\rho A^c_\sigma} A^d_\alpha A^e_\beta A^f_\gamma | 0 \rangle$$

$$\simeq \quad \langle 0 | \left(g f^{abc} g^{\mu\rho} g^{\nu\sigma} \partial_\mu A^a_\nu A^b_\rho A^c_\sigma \right) A^d_\alpha A^e_\beta A^f_\gamma | 0 \rangle$$

Find all possible contractions: $\quad A^d_\alpha A^b_\rho \rightarrow \delta^{bd} g_{\rho\beta} \; \cancel{G_F(x-y)}$ amputated

$$A^d_\alpha \partial_\mu A^a_\nu \rightarrow -i p_\mu \delta^{ad} g_{\nu\alpha} \; \cancel{G_F(x-y)} \quad \text{amputated}$$

– Each of the three external fields can contract into the $(\partial_\mu A^a_\nu)$. For each such contraction, there are two possible contractions among the other two external fields.

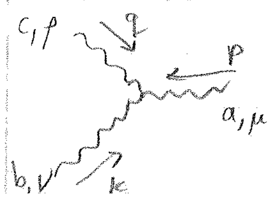
$$\text{Feynman Rule} = \underset{\text{For being a Feynman rule}}{i} \times g f^{abc} g^{\mu\rho} g^{\nu\sigma} \times \Big[-i p_\mu \delta^{ad} g_{\nu\alpha} \left(\delta^{be} g_{\rho\beta} \delta^{cf} g_{\sigma\gamma} + \delta^{bf} g_{\rho\gamma} \delta^{ce} g_{\sigma\beta} \right)$$
$$- i k_\mu \delta^{ae} g_{\nu\beta} \left(\delta^{bd} g_{\rho\alpha} \delta^{cf} g_{\sigma\gamma} + \delta^{bf} g_{\rho\gamma} \delta^{cd} g_{\sigma\alpha} \right)$$
$$- i q_\mu \delta^{af} g_{\nu\gamma} \left(\delta^{bd} g_{\rho\alpha} \delta^{ce} g_{\sigma\beta} + \delta^{be} g_{\rho\beta} \delta^{cd} g_{\sigma\alpha} \right) \Big]$$

$$= + g \Big(f^{def} p_\beta g_{\gamma\alpha} + f^{dfe} p_\gamma g_{\beta\alpha} + f^{edf} k_\alpha g_{\gamma\beta} + f^{efd} k_\gamma g_{\alpha\beta}$$
$$+ f^{fde} q_\alpha g_{\beta\gamma} + f^{fed} q_\beta g_{\alpha\gamma} \Big)$$

$$= g f^{def} \left(g_{\alpha\beta} (k-p)_\gamma + g_{\beta\gamma} (q-k)_\alpha + g_{\gamma\alpha} (p-q)_\beta \right)$$

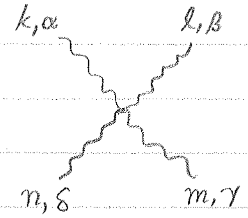
Clean up:

combinatoric factor = 1/6



$$= -g f^{abc} \left(g_{\mu\nu} (p-k)_\rho + g_{\nu\rho} (k-q)_\mu + g_{\rho\mu} (q-p)_\nu \right)$$

Feynman Rule for $-\frac{g^2}{4} f^{abc} f^{cde} A^a_\mu A^b_\nu A^{\mu d} A^{\nu e}$



$$\sim \langle 0 | \left(\frac{-g^2}{4} f^{abc} f^{cde} A^a_\mu A^b_\nu A^{\mu c} A^{\nu d} \right) A^k_\alpha A^l_\beta A^m_\gamma A^n_\delta | 0 \rangle$$

$$= \langle 0 | \left(\frac{-g^2}{4} f^{abc} f^{cde} g^{\mu\rho} g^{\nu\sigma} A^a_\mu A^b_\nu A^d_\rho A^e_\sigma \right) A^k_\alpha A^l_\beta A^m_\gamma A^n_\delta | 0 \rangle$$

Find all contractions. Use: $A^a_\mu A^k_\alpha \longrightarrow \delta^{ak} g_{\mu\alpha} G_F(x-y)$

Each external field can contract into any of the vertex fields $\Longrightarrow$ 24 different contraction.

<u>NOTATION</u> to handle permutations:

e.g.: $-\frac{g^2}{4} f^{abc} f^{cde} g^{\mu\rho} g^{\nu\sigma} \langle 0 | \left(A^a_\mu A^b_\nu A^d_\rho A^e_\sigma \right) A^k_\alpha A^l_\beta A^m_\gamma A^n_\delta | 0 \rangle$

(1) (2) (3) (4) ← Number ext fields

$$= -\frac{g^2}{4} f^{abc} f^{cde} g^{\mu\rho} g^{\nu\sigma} \left(\delta^{ak} \delta^{bl} \delta^{dm} \delta^{en} g_{\mu\alpha} g_{\nu\beta} g_{\rho\gamma} g_{\sigma\delta} \right)$$

$$= -\frac{g^2}{4} f^{klc} f^{cmn} g_{\alpha\gamma} g_{\beta\delta} \quad \equiv \quad -\frac{g^2}{4} f^{12x} f^{x34} g_{13} g_{24}$$

Numbers indicate which field has contracted where.

Simplify further: Given the first tensor f^{12x}, we can deduce the next tensor: f^{x34}
similarly, given the first metric tensor, g_{13}, the next one must be g_{24}.

$\Longrightarrow$ DROP second factor

$\downarrow$

<u>Note on ordering</u>: Choose the convention that the antisymmetric tensor must be put in proper numerical order — from smallest to greatest.

examples: $f^{21x} f^{x34} g_{23} g_{14} \longrightarrow -F^{12} G_{14}$

swap these indices to put in proper order

$f^{23x} f^{x14} g_{13} g_{24} \xrightarrow{\text{OK}} F^{14} G_{13}$

$f^{31x} f^{x42} g_{12} g_{34} \longrightarrow F^{13} G_{12}$

Only display tensors with "1" on them.

Proceed to write out all contractions:

$$\begin{matrix}\text{Feynman}\\ \text{Rule}\end{matrix} = i\,\frac{-g^2}{4}\left[f^{12x}f^{x34}\,g_{13}\,g_{24} + \begin{matrix}\text{All permutations}\\ \text{of } 1,2,3,4\end{matrix}\right]$$

$$= \frac{-ig^2}{4}\Big[F^{12}G_{13} + (1\leftrightarrow2) + (1\leftrightarrow3) + (1\leftrightarrow4) + (2\leftrightarrow3) + (2\leftrightarrow4) + (3\leftrightarrow4)$$

$$+ \begin{pmatrix}1\leftrightarrow2\\ 3\leftrightarrow4\end{pmatrix} + \begin{pmatrix}1\leftrightarrow3\\ 2\leftrightarrow4\end{pmatrix} + \begin{pmatrix}1\leftrightarrow4\\ 2\leftrightarrow3\end{pmatrix}$$

$$+ (1\to2\to3\to1) + (1\to3\to2\to1) + (1\to2\to4\to1) + (1\to4\to2\to1) + (1\to3\to4\to1) + (1\to4\to3\to1) + (2\to3\to4\to2) + (2\to4\to3\to2)$$

$$+ (1\to2\to3\to4\to1) + (1\to2\to4\to3\to1) + (1\to3\to2\to4\to1) + (1\to3\to4\to2\to1) + (1\to4\to2\to3\to1) + (1\to4\to3\to2\to1)\Big]$$

$$= \frac{-ig^2}{4}\Big[F^{12}G_{13} - F^{12}G_{14} - F^{14}G_{13} + F^{13}G_{12} + F^{13}G_{12} - F^{14}G_{13} - F^{12}G_{14}$$

$$+ F^{12}G_{13} + F^{12}G_{13} + F^{12}G_{13}$$

$$+ F^{14}G_{12} - F^{13}G_{14} - F^{13}G_{14} + F^{14}G_{12} + F^{14}G_{12} - F^{13}F_{14} - F^{13}G_{14} + F^{14}G_{12}$$

$$- F^{14}G_{13} + F^{13}G_{12} - F^{12}G_{14} + F^{13}G_{12} - F^{12}G_{14} - F^{14}G_{13}\Big]$$

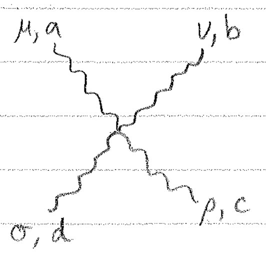
Combine terms:

$$\begin{matrix}\text{Feynman}\\ \text{Rule}\end{matrix} = \frac{-ig^2}{4}\Big[4F^{12}(G_{13}-G_{14}) + 4F^{13}(G_{12}-G_{14}) + 4F^{14}(G_{12}-G_{13})\Big]$$

Unpacking the notation,

$$= -ig^2\Big[f^{12x}f^{x34}(g_{13}\,g_{24} - g_{14}\,g_{23}) + f^{13x}f^{x24}(g_{12}\,g_{34} - g_{14}\,g_{23}) + f^{14x}f^{x23}(g_{12}\,g_{34} - g_{13}\,g_{24}$$

Now put in all the indices, (clean up)

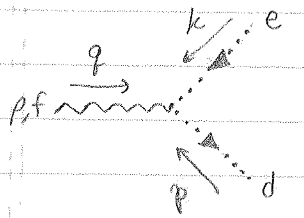


$$= -ig^2\Big[f^{abe}f^{ecd}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{ebd}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho})$$

$$+ f^{ade}f^{ebc}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\Big]$$

combinatoric factor $= \dfrac{1}{24}$

Feynman rule for $+g f^{abc}(\partial_\mu \eta^a)^\dagger \eta^b A^{\mu c}$ (Ghost-gluon vertex)



$$\sim \langle 0|\, g f^{abc} (\partial_\mu \eta^a)^\dagger \eta^b A^{\mu c} \,|\, \bar{\eta}_d(p)\, \eta_e(k)\, A(q) \rangle$$

$$\langle 0|\, g\, g^{\mu\nu} f^{abc} (\partial_\mu \eta^a)^\dagger \eta^b A^c_\nu \quad \eta^d \eta^{\dagger e} A^f_\rho \,|0\rangle$$

Only one possible contraction

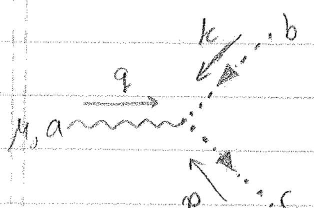
(shown) $\quad A^c_\nu A^f_\rho \longrightarrow \delta^{cf} g_{\nu\rho}$ ~~$\delta(x-y)$~~ amp.

$$\eta^{a\dagger} \eta^d \longrightarrow \delta^{ae}$$

Feynman Rule $= i \times g\, g^{\mu\nu} f^{abc} (-i p_\mu \delta^{ad})\, \delta^{be} \delta^{cf} g_{\nu\rho}$

$$= g f^{def} p_\rho = -g f^{fed} p_\rho$$

Clean up:



$= -g f^{abc} p_\mu$

Notice: derivative always on antighost

$\Rightarrow$ Feynman rule: momentum of antighost.

Seems asymmetric?

Ghosts enter into diagrams as loops.

Thus ghost momentum and antighost momentum get counted.

e.g.: