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Summary

— continuum normalization —

(non. relativistic norm)

$$\hat{H}_{int} = \frac{-e}{m}\vec{A}\cdot\vec{p} - \vec{\mu}\cdot(\vec{\nabla}\times\vec{A}) + \frac{e^2}{2m}\vec{A}^2$$

$$= \int_0^\infty dk \sum_{J,M}\left(\hat{a}^{\mathcal{E}}_{kJM}\hat{H}^{\mathcal{E}}_{kJM} + \hat{a}^{\mathcal{M}}_{kJM}\hat{H}^{\mathcal{M}}_{kJM} + h.c.\right)$$

[Absorption displayed]

$$\hat{H}^{\mathcal{E}\text{ or }\mathcal{M}}_{kJM} = \underbrace{(-i)}_{\text{only for }\mathcal{M}}\, i^J\sqrt{\frac{2\hbar c(J+1)k^{2J+1}}{\pi J(2J+1)!!^2}}\,\frac{1}{\sqrt{4\pi\epsilon_0}}\,\hat{\mathcal{O}}^{\mathcal{E}\text{ or }\mathcal{M}}_{J,M}$$

$$\hat{\mathcal{O}}^{\mathcal{E}}_{JM} = e\, r^J\, Y^M_J(\Omega) + \dots$$

corrections to Siegert's theorem.

$$\hat{\mathcal{O}}^{\mathcal{M}}_{JM} = \frac{e}{2mc}\left(\frac{2}{J+1}\vec{L} + g\vec{S}\right)\cdot\underbrace{\vec{\nabla}\left(r^J Y^M_J(\Omega)\right)}_{\sqrt{J(2J+1)}\, r^{J-1}\,\vec{Y}^{J-1}_{J,M}(\Omega)}$$

Case $J=1$: Dipole transition (absorption)

$$\hat{H}^{\mathcal{E}\text{ or }\mathcal{M}}_{k1M} = \underbrace{(-i)}_{\text{for }\mathcal{M}}\, i\sqrt{\frac{\hbar c k^3}{\pi}}\,\frac{2}{3}\,\frac{1}{\sqrt{4\pi\epsilon_0}}\,\hat{\mathcal{O}}^{\mathcal{E}\text{ or }\mathcal{M}}_{1,M}$$

spherical basis:

$$\hat{\mathcal{O}}^{\mathcal{E}}_{1,M} = e\; r\; Y^M_1(\Omega)$$

$$\hat{\mathcal{O}}^{\mathcal{M}}_{1,M} = \frac{1}{2mc}\, e\left(\vec{L} + g\vec{S}\right)\cdot\sqrt{\frac{3}{4\pi}}\,\vec{\varepsilon}_{\lambda=M}$$

cartesian basis:

$$\vec{\hat{\mathcal{O}}}^{\mathcal{E}} = e\sqrt{\frac{3}{4\pi}}\,\vec{x} = \sqrt{\frac{3}{4\pi}}\,\vec{d}$$

$$\vec{\hat{\mathcal{O}}}^{\mathcal{M}} \overset{(?)}{=} \frac{e}{2mc}\left(\vec{L} + g\vec{S}\right)$$