

Selection rule for E1 transitions

GENERALLY: Angular momentum selection

Wigner 3J:  $\begin{pmatrix} j & 1 & j' \\ m_j & M & -m_{j'} \end{pmatrix}$  ← Must satisfy triangle inequality  
 ← Must satisfy addition law

$$|j-1| \leq j' \leq |j+1| \Rightarrow \Delta j = \{0, \pm 1\}$$

$$|j'-j| \leq 1 \leq |j'+j| \Rightarrow j=0 \nrightarrow j'=0$$

$$m_j + M - m_{j'} = 0$$

Parity

$$C_{l0,10}^{l'0} \leftarrow l \neq l' \quad (\text{Orbital angular momentum})$$

More specifically, must flip parity.

If system is LS - (Russell) - Saunders coupled :

- valid for light atomic systems (non-relativistic)
- and other systems where non-central couplings are unimportant.

Wigner 6J symbol:

$$\begin{Bmatrix} l & s & j \\ j' & 1 & l' \end{Bmatrix}$$

triad triangularity conditions

$$|l-1| \leq l' \leq |l+1| \Rightarrow \Delta l = \{0, \pm 1\}$$

$$|l'-l| \leq 1 \leq |l'+l| \Rightarrow l=0 \nrightarrow l'=0 \quad \text{forbidden by parity selection anyway.}$$

$$|j-1| \leq j' \leq |j+1| \Rightarrow \Delta j = \{0, \pm 1\}$$

$$|j'-j| \leq 1 \leq |j'+j| \Rightarrow j'=0 \nrightarrow j=0$$

$$|l-s| \leq j \leq |l+s| \quad (\text{obvious})$$

$$|l'-s| \leq j' \leq |l'+s| \quad [\text{trivial consequence of spin selection}]$$

$$S_{SS'} \Rightarrow \text{Spin multiplicity unchanged.}$$