Cases:

$\underline{\omega = \omega'}$ (Elastic scattering)

① If $\underset{\mathcal{O}(100\,\text{eV})}{\omega} \gg \underbrace{|E_{A_{i\,\text{or}\,f}} - E_{A_n}|}_{\mathcal{O}(\text{eV})}$ (photon frequency much larger than atomic frequency differences)

Can neglect denominators [is dipole approx still valid?]

$$\frac{d\sigma}{d\Omega} \approx \frac{\alpha c}{m\hbar} \underset{1}{\left(\frac{\omega'}{\omega}\right)} \left| N_e\, \vec{\varepsilon}'^{*} \cdot \vec{\varepsilon} + \mathcal{O}\left(\frac{1}{\omega}\right) \right|^2$$

$$= \left(\frac{\alpha c}{m\hbar}\right) N_e^2 \left|\vec{\varepsilon}'^{*}(\hat{k}') \cdot \vec{\varepsilon}(\hat{k})\right|^2$$

How high must ω be so that $\lambda \approx a_0$

$\omega = ck = c\frac{2\pi}{\lambda} = 23\ \text{keV}/\hbar$

[atomic transition frequencies $\mathcal{O}(\text{eV})$

so yes! still valid.

photon resolves (temporally) electrons, and scatters off them

[Thompson scattering]

② Low energy: Rayleigh scattering.

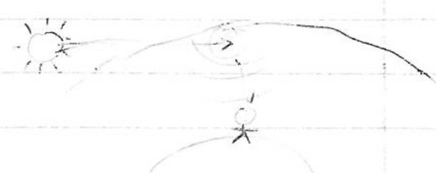
Why the sky is blue:

If $\omega \ll |E_{A_{i\,\text{or}\,f}} - E_{A_n}|$, use dipole form, and neglect widths and ω in denom

$$\frac{d\sigma}{d\Omega} \approx m^2 \left(\frac{\alpha\hbar}{mc}\right)^2 \omega^4 \left| \sum_n \frac{\vec{\varepsilon}'^{*}\cdot\vec{r}_{fn}\ \vec{\varepsilon}\cdot\vec{r}_{ni}}{E_{A_i} - E_{A_n}} + \frac{\vec{\varepsilon}\cdot\vec{r}_{fn}\ \vec{\varepsilon}'^{*}\cdot\vec{r}_{ni}}{E_{A_i} - E_{A_n}} \right|^2$$

$$\equiv m^2 \left(\frac{\alpha\hbar}{mc}\right)^2 \omega^4 \left| \sum_n \left(\vec{\varepsilon}'^{*}\cdot\vec{r}_{fn}\ \vec{\varepsilon}\cdot\vec{r}_{ni} + \vec{\varepsilon}\cdot\vec{r}_{fn}\ \vec{\varepsilon}'^{*}\cdot\vec{r}_{ni} \right) \frac{1}{E_{A_i} - E_{A_n}} \right|^2$$

- not suppressed by mass
- higher frequencies are most strongly scattered.



among the colors of the visible spectrum,

violet is most scattered, then blue, then green ... red scattered least

But this is also most attenuated — lost from spectrum

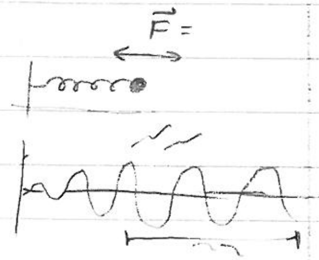
Thus the red colored sun at dawn.

Resonant scattering (Fluorenscence)

Kramers – Heisenberg formula diverges as $\hbar\omega \to E_n - E_i$.

Damping effects not included

Model: Driven oscillator with damping.



$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = a \cos \omega t$$

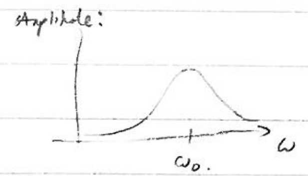
For long times, (steady state)

$$x(t) = \underbrace{\frac{a}{\sqrt{(\omega_0^2-\omega^2)^2 + \omega^2\gamma^2}}}_{\text{amplitude}} \cos\Big(\omega t - \underbrace{\tan^{-1}\frac{\omega\gamma}{\omega_0^2-\omega^2}}_{\text{phase shift.}}\Big)$$

On resonace

when $\omega = \omega_0$,

$$x(t) = \frac{a}{\omega\gamma} \cos(\omega t - 90^\circ)$$



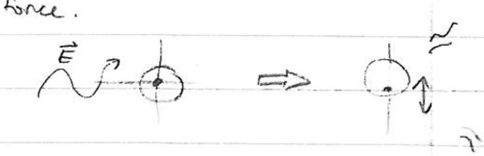
Note: on resonance amplitude is 90° out of phase with driving force

Applied to atomic system:

Electric component of EM field is the driving force.

The electron is like an oscillator

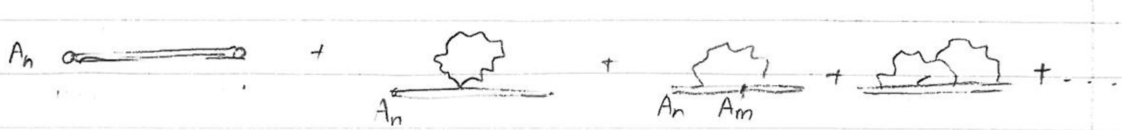
The emitted EM radiation is the damping.



Expect time dependence (after external field is turned off) $\sim e^{-t/2\tau_n}$ $\quad \tau$ = lifetime of state n

Mathematics:

Need to dress internal atomic line propagator:



→ to propagator methods