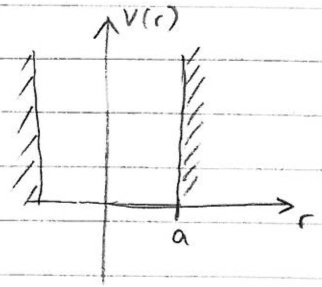
Bound states of infinite spherical potential



Schrödinger equation:

$$\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{\ell(\ell+1)}{r^2}\right] R(r) = \frac{2mE}{\hbar^2} E\, R(r)$$

Define $k^2 = \frac{2mE}{\hbar^2}$

Inside well, solution must be regular $\Rightarrow$ $R_\ell(r) = \mathcal{N} j_\ell(kr)$,

Subject to boundary condition $\underbrace{R_\ell(r=a) = 0.}$

Boundary condition imposes quantization condition on k (and, hence) on E.

Quantization condition: $j_\ell(ka) = 0$

Mathematica gives zeros of $J_n(x)$ $\longrightarrow$ related by $j_n(x) = \sqrt{\frac{\pi}{2x}}\, J_{n+\frac{1}{2}}(x)$

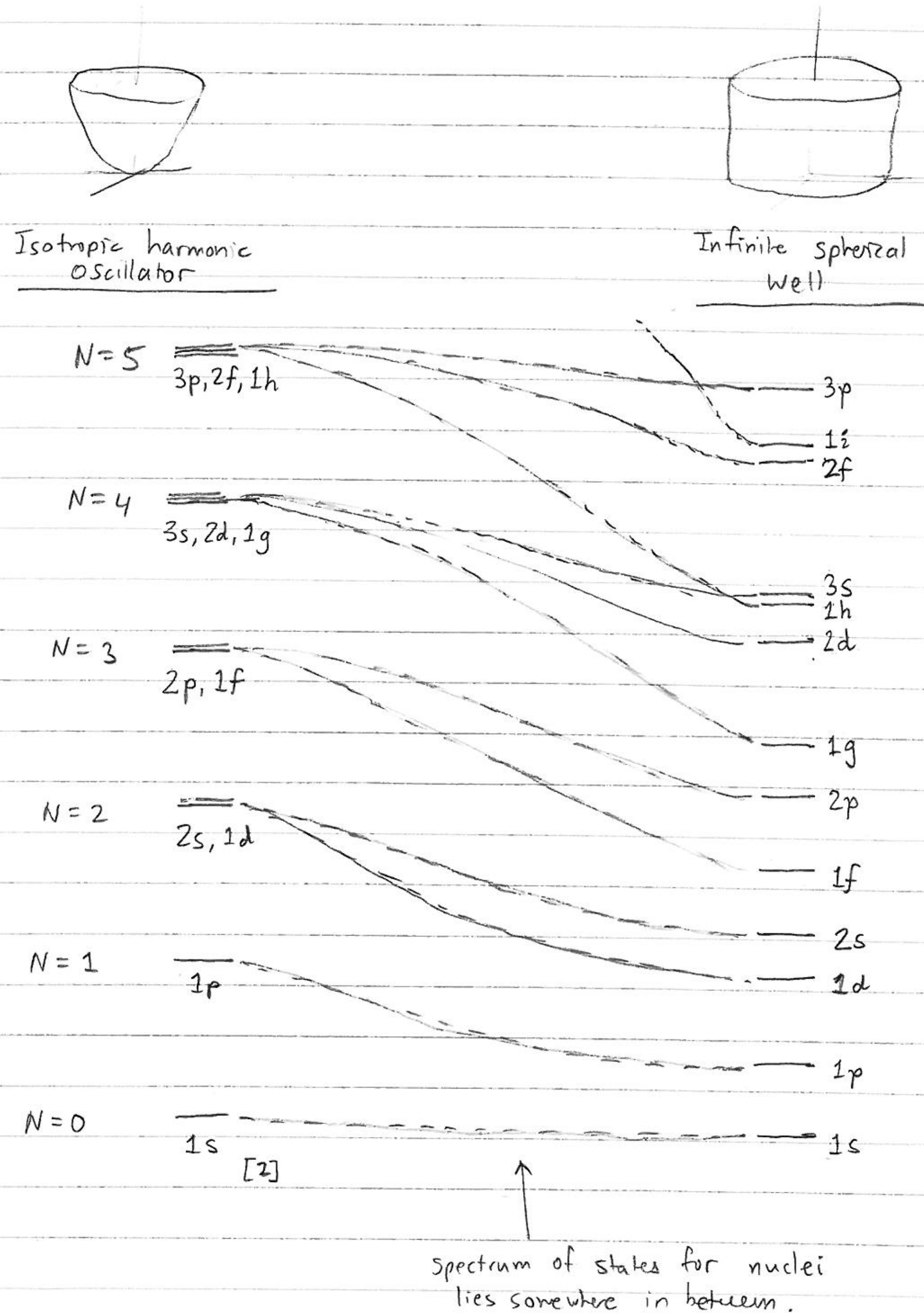
n^{th} zero of $j_\ell(ka)$ is $j^{(Root)}_{\ell+\frac{1}{2},n}$ = BesselJZero$[\ell+\frac{1}{2}, n]$.

Then $j^{(Root)}_{\ell+\frac{1}{2},n} = ka = \sqrt{\frac{2mE}{\hbar^2}}\, a$

$$\Rightarrow \boxed{E_{n,\ell} = \frac{\hbar^2}{2ma^2}\left(j^{(Root)}_{\ell+\frac{1}{2},n}\right)^2}$$

$\psi_{n,\ell,m} = \mathcal{N} j_\ell(kr)\, Y_\ell^m(\theta,\phi)$

To be determined numerically.


Isotropic harmonic oscillator
Infinite spherical well
N=5
3p, 2f, 1h
N=4
3s, 2d, 1g
N=3
2p, 1f
N=2
2s, 1d
N=1
1p
N=0
1s
[2]
3p
1i
2f
3s
1h
2d
1g
2p
1f
2s
1d
1p
1s
spectrum of states for nuclei
lies somewhere in between.