Parity invariance of Dirac equation

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{x},t) = \left(c\vec{\alpha}\cdot\vec{p} + \beta mc^2\right)\psi(\vec{x},t)$$

Try: under parity $\vec{x} \longrightarrow -\vec{x}$. $\vec{p} \to -\vec{p}$

$$i\hbar \frac{\partial}{\partial t}\psi(-\vec{x},t) = \left(c\vec{\alpha}\cdot(-\vec{p}) + \beta mc^2\right)\psi(-\vec{x},t)$$

Equation slightly different.
Multiply at left by β:

$$i\hbar \frac{\partial}{\partial t}\left[\beta\psi(-\vec{x},t)\right] = \left(c\,\beta\vec{\alpha}\cdot(-\vec{p}) + \beta\beta mc^2\right)\psi(-\vec{x},t)$$

$\ominus$ $\qquad \{\vec{\alpha},\beta\} = 0.$

$$= \left(c\vec{\alpha}\cdot\vec{p}\,\beta + \beta\beta mc^2\right)\psi(-\vec{x},t)$$

$$i\hbar \frac{\partial}{\partial t}\left[\beta\psi(-\vec{x},t)\right] = \left(c\vec{\alpha}\cdot\vec{p} + \beta mc^2\right)\left[\beta\psi(-\vec{x},t)\right]$$

$\therefore$ Dirac equation is parity invariant provided:

$$\psi(\vec{x},t) \xrightarrow{P} \eta_p\,\beta\,\psi(-\vec{x},t).$$

<u>Intrinsic Parity</u>
convention: $\eta_p = \pm 1$ (old)
$\eta_p = -i$ (new) for QFT

$$= \eta_p \begin{pmatrix} \psi_1(-\vec{x},t) \\ \psi_2(-\vec{x},t) \\ -\psi_3(-\vec{x},t) \\ -\psi_4(-\vec{x},t) \end{pmatrix}$$

Under parity, the lower two ("small") components have opposite <u>intrinsic parity</u>.

If $\psi(\vec{x},t)$ solves the Dirac equation, then so does $\psi^P(\vec{x},t) = \beta\psi(-\vec{x},t)$.