

Quadratic Form of Dirac equation

$e \rightarrow -|e|$ for electron.

Dirac equation:

$$[i\hbar \gamma^\mu D_\mu - mc] \psi = 0 \quad D_\mu = \partial_\mu + \frac{ie}{\hbar} A_\mu$$

left-multiply by $(i\hbar \gamma^\nu D_\nu + mc)$

$$(i\hbar \gamma^\mu D_\mu + mc)(i\hbar \gamma^\nu D_\nu - mc) \psi = 0.$$

$$\therefore \left(-\hbar^2 \underbrace{\gamma^\mu \gamma^\nu D_\mu D_\nu}_{\text{Symmetrize:}} - \cancel{i\hbar mc \gamma^\mu D_\mu} + \cancel{i\hbar mc \gamma^\nu D_\nu} - m^2 c^2 \right) \psi = 0.$$

$$\begin{aligned} &= \left(\frac{1}{2} \{ \gamma^\mu, \gamma^\nu \} + \frac{1}{2} [\gamma^\mu, \gamma^\nu] \right) \left(\frac{1}{2} \{ D_\mu, D_\nu \} + \frac{1}{2} [D_\mu, D_\nu] \right) \\ &= \frac{1}{4} \underbrace{\{ \gamma^\mu, \gamma^\nu \}}_{2g^{\mu\nu} \Rightarrow 2D^2} \{ D^\mu, D^\nu \} + \frac{1}{4} \underbrace{[\gamma^\mu, \gamma^\nu]}_{-2i\sigma^{\mu\nu}} \underbrace{[D_\mu, D_\nu]}_{\frac{ie}{\hbar} F_{\mu\nu}} \\ &= D^2 + \frac{e}{2\hbar} \sigma^{\mu\nu} F_{\mu\nu} \end{aligned}$$

$$\therefore \underline{\underline{[-\hbar^2 D^2 + \frac{e}{2\hbar} \sigma^{\mu\nu} F_{\mu\nu} - m^2 c^2] \psi = 0}}$$