## Time-dependent perturbation theory - interaction picture

Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{H}'$

$\hat{H}_0 \equiv$ exactly solvable part

$\hat{H}'(t) \equiv$ perturbations (possibly explicitly time-dependent).

Want to compute the probability amplitude for state $\psi_i(t_i)$ at time $t_i$ to evolve to state $\psi_f(t_f)$:

$$\langle \psi_f(t_f) | \psi_i(t_i) \rangle = \langle \psi_f | \hat{U}(t_f, t_i) | \psi_i \rangle$$

express in terms of interaction picture time evolution operator

$$= \langle \psi_f | \hat{U}_0(t_f)\, \hat{U}_I(t_f, t_i)\, \hat{U}_0^\dagger(t_i) | \psi_i \rangle$$

For simplicity*, consider transitions between distinct e-states of $H_0$:

$$\hat{H}_0 |i\rangle = E_i |i\rangle \qquad \hat{H} |f\rangle = E_f |f\rangle \qquad \langle f | i \rangle = 0 \quad \text{(orthogonal)}$$

$$\langle f(t_f) | i(t_i) \rangle = \langle f | \hat{U}_0(t_f)\, \hat{U}_I(t_f, t_i)\, \hat{U}_0^\dagger(t_i) | i \rangle$$

$$= e^{-i(E_f t_f - E_i t_i)/\hbar} \langle f | \hat{U}_I(t_f, t_i) | i \rangle$$

$$= e^{-i(E_f t_f - E_i t_i)/\hbar} \langle f | T\left[ 1 - \frac{i}{\hbar} \int_{t_i}^{t_f} dt'\, \hat{H}_I'(t') + \dots \right] | i \rangle$$

Heisenberg states

Perturbation Hamiltonian in interaction picture, with possible explicit time dependence.

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* justified by the asymptotic condition together with the adiabatic hypothesis that $\hat{H}'$ is inoperative long before and after the interaction.