

In d -dimensions

Wavepacket: $|\psi(t=0)\rangle$

$$\langle \vec{x} | \psi(0) \rangle = \psi(\mathbf{x}, 0) = \mathcal{N} \exp \left[-\frac{(\vec{x} - \vec{x}_0)^2}{4(\Delta x)^2} + i\vec{k}_0 \cdot \vec{x} + i\phi \right]$$

where $\mathcal{N} = (2\pi\Delta x^2)^{-d/4}$

Moments:

$$\begin{aligned} \langle \vec{x} \rangle &= \vec{x}_0 & \langle \vec{p} \rangle &= \hbar \vec{k}_0 \\ \langle \vec{x} \cdot \vec{x} \rangle &= d(\Delta x)^2 + \vec{x}_0^2 & \langle p_i^2 \rangle &= \frac{\hbar^2}{4\Delta x^2} + \hbar(k_0)_i^2 \\ \Delta r &\equiv \sqrt{\langle \vec{x} \cdot \vec{x} \rangle - \langle \vec{x} \rangle \langle \vec{x} \rangle} = \sqrt{d}\Delta x & \Delta p_i &\equiv \sqrt{\langle p_i^2 \rangle - \langle p_i \rangle^2} = \frac{\hbar}{2} \frac{1}{\Delta x} \\ \langle x_i^2 \rangle &= \Delta x^2 + (x_0)_i^2 \end{aligned}$$

Wavepacket in momentum space:

$$\langle \vec{k} | \psi(0) \rangle = \psi(\mathbf{x}, 0) = \mathcal{N}_k \exp \left[-\frac{(\vec{k} - \vec{k}_0)^2}{4(\Delta k)^2} - i(\vec{k} - \vec{k}_0) \cdot \vec{x}_0 + i\phi \right]$$

where $\mathcal{N}_k = (2\pi\Delta k^2)^{-d/4}$

In 3-dimensions

Partial wave series of wavepacket: Assuming \vec{k}_0 collinear to \vec{x}_0 ,

$$\begin{aligned} \langle \vec{x} | \psi(0) \rangle &= \int_0^\infty dk \sum_{\ell, m} \langle \vec{x} | k\ell m \rangle \langle k\ell m | \psi(0) \rangle \\ &= \mathcal{N} e^{-\frac{(r^2 - x_0^2)}{4\Delta x^2} + i\phi} \sum_{\ell} i^\ell (2\ell + 1) j_\ell(k_0 r - \frac{i x_0 r}{2(\Delta x)^2}) P_\ell(\cos \theta) \end{aligned}$$

with partial wave expansion coefficient

$$\begin{aligned} \langle k\ell m | \psi(0) \rangle &= \psi_{k\ell m}(t=0) \\ &= \mathcal{N}_k i^\ell k \sqrt{4\pi(2\ell + 1)} e^{-\frac{k^2 + k_0^2}{4\Delta k^2}} j_\ell \left(-k \left(x_0 + \frac{i k_0}{2\Delta k^2} \right) \right) \delta_{m,0} e^{+i k_0 x_0 + i\phi} \end{aligned}$$