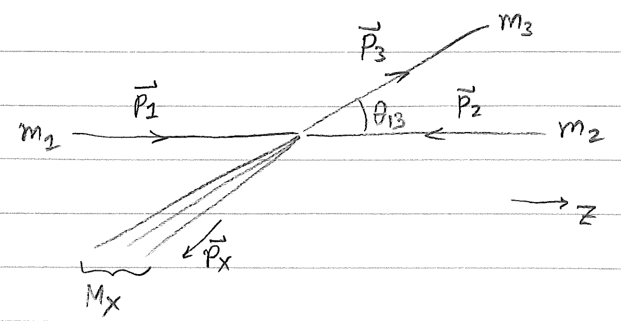
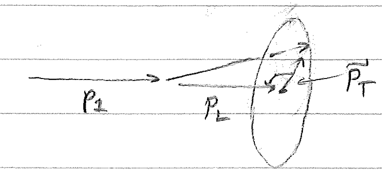
Inclusive process kinematics $\quad 1+2 \rightarrow 3+X$





COM system:

$$p_1 = (E_1;\, 0,0,p_z) \qquad p_1^2 = E_1^2 - p_z^2 = m_1^2 \qquad \Big(\sum_X p_X\Big)^2 = M_X^2$$

"missing mass"

$$p_2 = (E_2;\, 0,0,-p_z) \qquad p_2^2 = E_2^2 - p_z^2 = m_2^2$$

$$p_3 = (E_3;\, \vec{p}_T,\, p_L) \qquad p_3^2 = E_3^2 - \vec{p}_T^{\,2} - p_L^2 = m_3^2$$

2-component transverse momentum

longitudinal momentum

$$E_3^2 - p_L^2 = \underbrace{m_3^2 + p_T^2}$$

Define: m_L^2 Longitudinal mass (effective mass of longitudinal part of momentum $E_3^2 = m_L^2 + p_L^2$)

Typical values

$m_3 \lesssim 1\text{ GeV}$ (particles of interest: $\pi^\pm, e^\pm, p^\pm$)

$|\vec{p}_T| \lesssim 0.5\text{ GeV}$ (mostly longitudinal)

$\Rightarrow \quad m_L \lesssim 1\text{ GeV}.$

Kinematic invariants – very similar to $2\rightarrow 2$ case

① $\quad s = (p_1+p_2)^2 = (p_3+p_X)^2$

$$\Rightarrow \quad E_1 = \frac{1}{2\sqrt{s}}(m_1^2 - m_2^2 + s) \xrightarrow{s\to\infty} \frac{\sqrt{s}}{2}$$

$$E_2 = \frac{1}{2\sqrt{s}}(-m_1^2 + m_2^2 + s) \longrightarrow \frac{\sqrt{s}}{2}$$

$$\Rightarrow \quad E_{COM} = E_1 + E_2 = \sqrt{s}$$

$$p_z^2 = \frac{1}{4s}\lambda(m_1^2, m_2^2, s) \longrightarrow \frac{s}{4}$$

($\equiv |\vec{p}_i|^2$ from before)

Final state energy & momentum same, but $m_4 \to M_X$.

$$|\vec{p}_3|^2 = |p_T^2| + p_L^2 = \frac{1}{4s}\lambda(m_3^2, M_X^2, s)$$

$$= \frac{1}{4s}\left[s-(m_3+M_X)^2\right]\left[s-(m_3-M_X)^2\right] \xrightarrow[M_X^2\to\infty]{s\to\infty} \frac{1}{4s}\left[s-M_X^2\right]^2 \xrightarrow{s\gg M_X^2} \frac{s}{4}$$

$\approx p_L^2$ since $|\vec{p}_T|^2$ small.

$$E_3 = \frac{1}{2\sqrt{s}}(m_3^2 - M_X^2 + s) \xrightarrow[M_X^2\to\infty]{s\to\infty} \frac{1}{2\sqrt{s}}(s-M_X^2) \xrightarrow{s\gg M_X^2} \frac{\sqrt{s}}{2}$$

② $t = (p_1-p_3)^2 = m_1^2 + m_3^2 - 2p_1\cdot p_3$

$$= m_1^2 + m_3^2 - 2(\underbrace{E_1}_{\sim\frac{\sqrt{s}}{2}} E_3 - \underbrace{p_z}_{\sim\frac{\sqrt{s}}{2}} p_L) \xrightarrow[M_X^2\to\infty]{s\to\infty} -\sqrt{s}\,(E_3 - p_L)$$

$$= -\sqrt{s}\,\frac{\overbrace{(E_3^2-p_L^2)}^{=m_L^2}}{\underbrace{E_3}_{\sim\frac{1}{2\sqrt{s}}(s-M_X^2)} + \underbrace{p_L}_{\sim\frac{1}{2\sqrt{s}}(s-M_X^2)}}$$

$$= \frac{-s\,m_L^2}{s-M_X^2}$$

③ $u = (p_2-p_3)^2 = m_2^2 + m_3^2 - 2p_2\cdot p_3$

$$= m_2^2 + m_3^2 - 2(E_2E_3 + p_z p_L) \xrightarrow[M_X^2\to\infty]{s\to\infty} -\sqrt{s}\,(E+p_L) \approx (s-M_X^2)$$

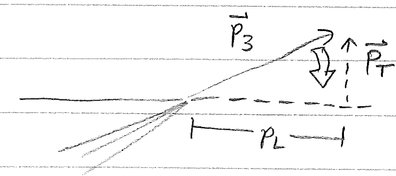
and of course, $s+t+u = m_1^2 + m_2^2 + m_3^2 + M_X^2$

$$\boxed{\text{Complete set of variables: } \underbrace{s}_{\text{set by experimenter}},\ \overbrace{t,\ M_X^2}^{\text{measured after reaction}}}$$

Another commonly used kinematic variable: Feynman x

—not to be confused with Bjorken x in DIS.

Feynman x: $x_F = \dfrac{p_L}{|p_L^{max}|}$



$p_L^{max} \equiv$ maximum possible longitudinal momentum for given s.

occurs when $\vec{p}_T = 0$ (t is as small as possible), also $M_X \to 0$.

$$(p_L^{max})^2 = \frac{1}{4s}\left[s-(m_3+M_X)^2\right]\left[s-(m_3-M_X)^2\right]\Big|_{M_X=0}$$

$$= \frac{1}{4s}(s+m_3^2)(s-m_3^2)$$

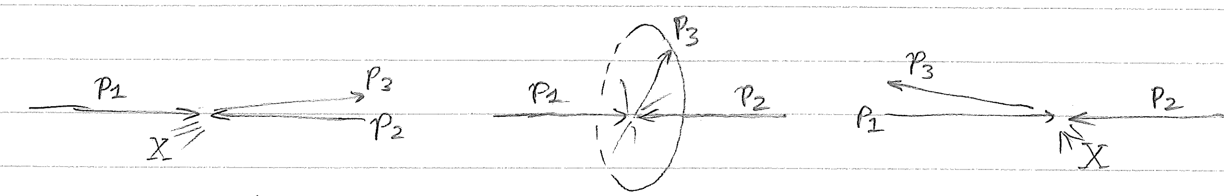
$$= \frac{1}{4s}(s^2-m_3^4) \xrightarrow{s\to\infty} \frac{s}{4}$$

$$x_F = \frac{\cos\theta_{cm}\,|\vec{p}_3|}{|p_L^{max}|}$$

So, $|x_F| \xrightarrow{s\to\infty} \dfrac{2p_L}{\sqrt{s}} \simeq \dfrac{2}{\sqrt{s}}\dfrac{1}{2\sqrt{s}}(s-M_X^2)$

$= 1 - \dfrac{M_X^2}{s}$ (large s limit)

Obviously $\boxed{-1 \le x_F < +1}$:



$x_F \approx 1$ — "particle 3 associated with 1"

$x_F \simeq 0$ — "particle 3 associated with neither 1 nor 2"

$x_F \approx -1$ — "particle 3 associated with 2"

Plug $s|x_F| \to s - M_X^2$ into $t \to \dfrac{-s\,m_L^2}{s-M_X^2}$

to get $t \longrightarrow \dfrac{-m_L^2}{x_F}$. ($x_F$ replaces t)

Complete set of variables: s, x_F, $|p_T|$