

3 → 3 unitarity equation (derivation)

$$\equiv (S) \equiv \oplus + \equiv (S) \equiv \oplus = \equiv \equiv$$

Write:  $\equiv \oplus \equiv \rightarrow - \equiv \oplus \equiv$        $\equiv \oplus \equiv \rightarrow - \equiv \ominus \equiv$

$$\equiv \oplus \equiv \rightarrow \equiv \equiv + \equiv \oplus \equiv + \equiv \oplus \equiv + \cancel{\equiv \oplus \equiv} + \equiv \oplus \equiv$$

$$\equiv \oplus \equiv \rightarrow \equiv \equiv - \equiv \ominus \equiv - \equiv \ominus \equiv - \cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv$$

So: Unitary eqn is

$$= - \equiv \oplus \ominus \equiv + (\equiv \equiv + \equiv \oplus \equiv + \equiv \oplus \equiv + \cancel{\equiv \oplus \equiv} + \equiv \oplus \equiv) (\equiv \equiv - \equiv \ominus \equiv - \equiv \ominus \equiv - \cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv)$$

cancel (2 particle unitary)

$$\begin{aligned} \equiv \equiv &= - \equiv \oplus \ominus \equiv + \equiv \equiv + (\cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv) + (\cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv) + (\cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv) + (\cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv) \\ &\quad - (\cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv) - (\cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv) - \equiv \oplus \ominus \equiv - \equiv \oplus \ominus \equiv \\ &\quad - (\cancel{\equiv \oplus \equiv} + \dots) \end{aligned}$$

cancel

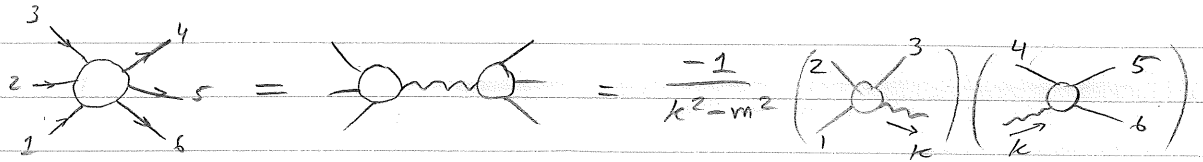
Solve for  $\equiv \oplus \equiv - \equiv \ominus \equiv$  :

$$\begin{aligned} \equiv \oplus \equiv - \equiv \ominus \equiv &= (\equiv \oplus \ominus \equiv + \equiv \oplus \ominus \equiv) + (\cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv + \cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv) \\ &\quad + (\cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv + \cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv) \\ &\quad + (\cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv + \cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv + \cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv + \cancel{\equiv \oplus \equiv} - \equiv \ominus \equiv) \end{aligned}$$

Factorization property of scattering amplitudes

So, what we have is

$$\begin{aligned} \text{Residue} &= i A_{123 \rightarrow 456} = i A_{12 \rightarrow 3k} \frac{i}{k^2 - m^2 + i\epsilon} i A_{k4 \rightarrow 56} + (\text{regular}) \\ \Rightarrow A_{123 \rightarrow 456} &= \frac{-1}{k^2 - m^2} A_{12 \rightarrow 3k} A_{k4 \rightarrow 56} + (\text{regular}) \end{aligned}$$



The amplitude "falls apart" into a product of two amplitudes at the pole.  
i.e. the residue of the pole factorizes into  $A_{12 \rightarrow 3k}$  &  $A_{k4 \rightarrow 56}$ .