Mandelstam Representation in relativistic scattering

Recall, single variable dispersion for $A(s,t)$ is s, for fixed t.

$$A(s,t) = \frac{g_s(t)}{m^2-s} + \frac{g_u(t)}{m^2-u} + \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{D_s(s',t)}{s'-s} + \frac{1}{\pi}\int_{u_{th}}^{\infty} du' \frac{D_u(u',t)}{u'-u}$$

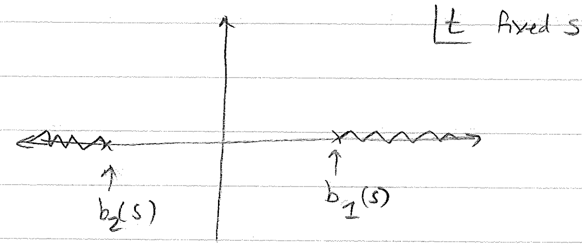
$$s+t+u = \sum_i m_i^2 = s'+t+u'$$

where $D_s(s',t) \equiv \frac{1}{2}\left[A(s'+i\epsilon,t) - A^*(s'+i\epsilon,t)\right]$

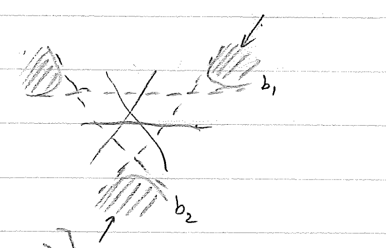
$$= \operatorname{Im} A(s'+i\epsilon, t) \qquad \text{for real } t.$$

But for imaginary t, cannot be written as an imaginary part, because
$A^*(s+i\epsilon,t) \neq A(s-i\epsilon,t)$ for complex t.

In the complex-t plane,
$D_s(s,t)$ has a cut dictated by
a Karplus curve.



Then, write a dispersion relation for $D_s(s,t)$, fixed s.

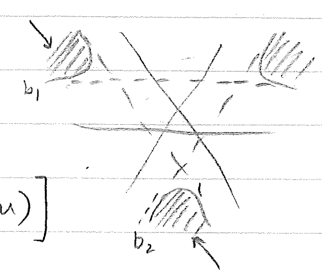


$$D_s(s,t) = \frac{1}{\pi}\int_{b_1}^{\infty} \frac{dt''}{t''-t}\,\frac{1}{2i}\left[D_s(s,t''+i\epsilon) - D_s(s,t''-i\epsilon)\right]$$

$$+ \frac{1}{\pi}\int_{b_2}^{\infty} \frac{du''}{u''-u}\,\frac{1}{2i}\left[D_s(s,u''+i\epsilon) - D_s(s,u''-i\epsilon)\right]$$

(could have written it as integral over t' from $-\infty \to b_2$ using $s+t+u = \sum m^2$)

Similar for $D_u(u,t)$, in complex t-plane, for fixed u.



$$D_u(u,t) = \frac{1}{\pi}\int_{b_1(u)}^{\infty} \frac{dt''}{t''-t}\,\frac{1}{2i}\left[D_u(u,t''+i\epsilon) - D_u(u,t''-i\epsilon)\right]$$

$$+ \frac{1}{\pi}\int_{b_2(u)}^{\infty} \frac{ds''}{s''-s}\,\frac{1}{2i}\left[D_s(s''+i\epsilon,u) - D_s(s''-i\epsilon,u)\right]$$

These are valid provided $D_s, D_u \to 0$ with $|t|\to\infty$ quickly enough,
and there are no additional poles in the complex t-plane.

Plugging these into the single-variable dispersion relation,

$$A(s,t) = \begin{pmatrix}\text{pole}\\ \text{terms}\end{pmatrix} + \frac{1}{\pi^2}\left[\iint \frac{ds'\,dt''}{(s'-s)(t''-t)}\rho_{st}(s',t'') + \iint \frac{ds'\,du''}{(s'-s)(u''-u)}\rho_{su}(s',u'')\right.$$

$$\left. + \iint \frac{du'\,dt''}{(u'-u)(t''-t)}\rho_{tu}(u',t'') + \iint \frac{du'\,ds''}{(u'-u)(s''-s)}\rho_{su}(s'',u')\right]$$

rename $u' \to u''$, $s'' \to s'$

Somehow (?) 2nd & 4th terms combine to give the Mandelstam representation.

$$A(s,t) = \begin{pmatrix}\text{pole}\\ \text{terms}\end{pmatrix} + \frac{1}{\pi^2}\left[\iint \frac{ds'\,dt'}{(s'-s)(t'-t)}\rho_{st}(s',t'') + \iint \frac{ds'\,du'}{(s'-s)(u'-u)}\rho_{su}(s',u')\right.$$

$$\left. + \iint \frac{du'\,dt'}{(u'-u)(t'-t)}\rho_{tu}(u',t')\right]$$

Integration is over the regions on which the double spectral functions ρ are defined.