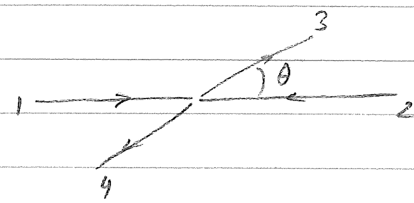
Partial wave decomposition

$1,2 \to 3,4$



for elastic scattering $1 \equiv 3, \quad 2 \equiv 4$.

Partial wave expansion:

$$A(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1)\, A_\ell(s)\, P_\ell(\cos\theta)$$

note $A(s,t) \equiv$ dimensionless.

Sometimes, one sees 16π here for simpler unitarity relations.

Inverse:

$$A_\ell(s) = \frac{1}{2}\int_{-1}^{1} d(\cos\theta)\, P_\ell(\cos\theta)\, A(s, t(\cos\theta))$$

and $\frac{1}{16\pi}$ here

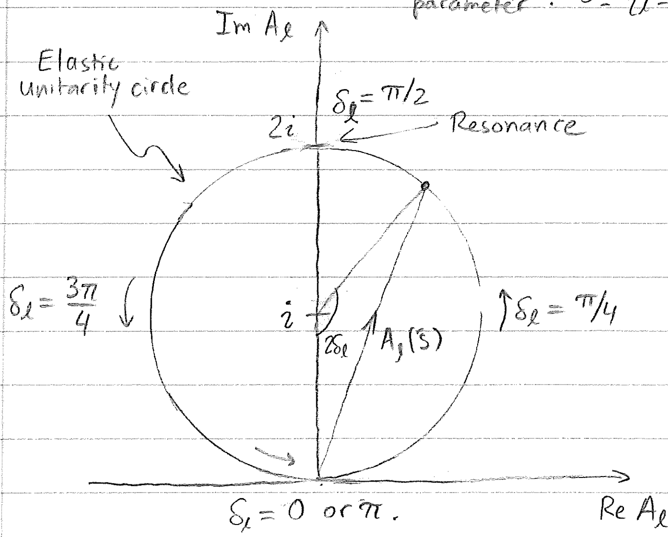
Then partial wave unitarity implies:

$$A_\ell(s) = \frac{8\pi i}{v}\left(1 - \eta_\ell\, e^{2i\delta_\ell(s)}\right)$$

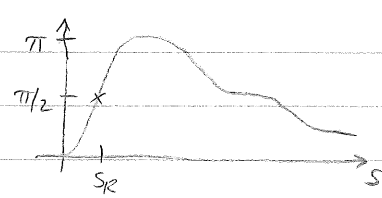
Phase shift

Inelasticity parameter: $0 \le \eta_\ell \le 1$

$v \equiv$ velocity of incoming particles (between 0 & 1).



As s increases, δ_ℓ goes up, and then down.



A_ℓ goes counter clockwise quickly, then counterclockwise to 0 slowly.

Argand diagram for partial waves.

(plotting expression in parenthesis)