Theoretical aside: karplus curve.

Consider now imaginary part of partial wave exp:

$$\text{Im}\, A(s,t) = \sum_{\ell} (2\ell+1)\, \text{Im}\, A_\ell(s)\, P_\ell(z) \qquad P_\ell(z) \in \mathbb{R} \text{ (just a poly).}$$

since $\text{Im}\, A_\ell(s) \sim |p|^{4\ell+1}$ falls faster than

$P_\ell(z) \sim \frac{1}{|p|^{2\ell}}$ grows,

the partial wave expansion for the imaginary part still converges beyond nearest t-channel threshold $t = 4m^2$ (especially near s-channel threshold)

It turns out large ℓ behavior of $P_\ell(z)$ would eventually cause partial wave expansion for $\text{Im}\, A_\ell(s)$ to fail:

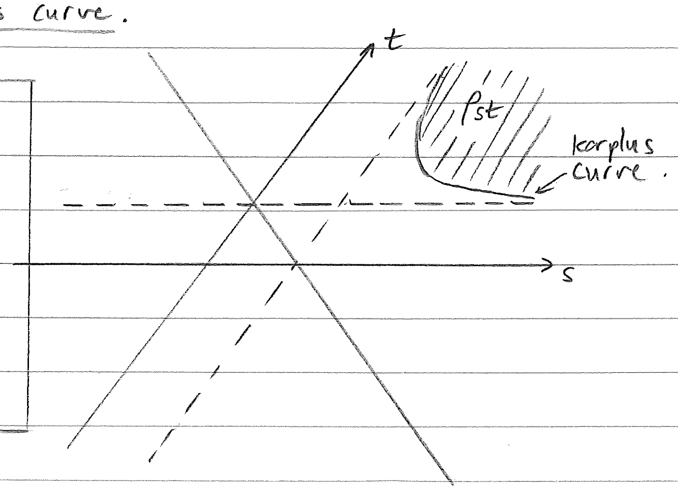
$$P_\ell(z) \sim \frac{e^{(\ell+1/2)\alpha}}{\sqrt{2\pi\ell}\, \sinh\alpha} \qquad \text{where} \quad \alpha = \cosh^{-1} z(t) > 1$$

This region is bounded by the karplus curve.

⇒ Implies that the discontinuity function

$$\text{Im}\, A(s,t) \equiv D_s(t)$$

itself, has a discontinuity!
(when viewed as an analytic function of t).



(karplus curve serves as the boundary for the double spectral function in the Mandelstam representation.