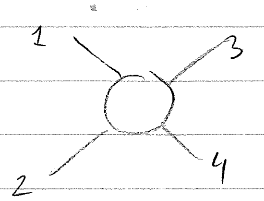
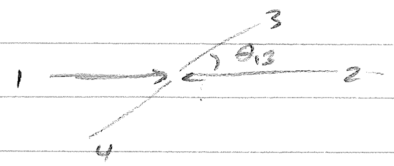
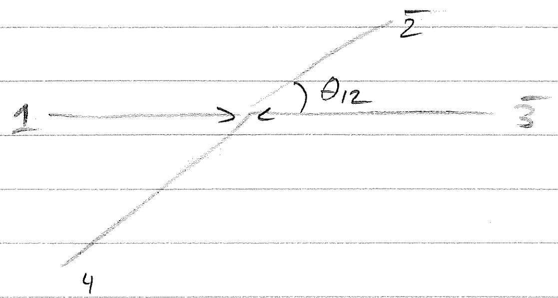
Orgin of signature



s-channel : $12 \to 34$



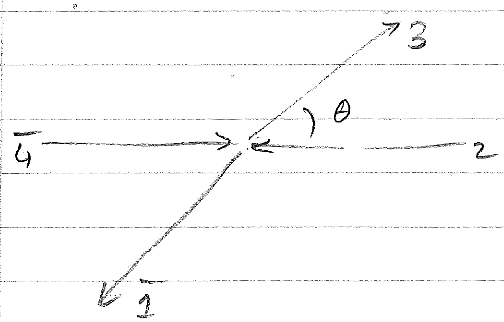
t-channel : $1\bar{3} \to \bar{2}4$



$\theta_{12} \equiv$ angular variable in t-channel
$\equiv$ energy variable in s-channel.

take $\theta_{12} \to \infty$ for Regge limit

u-channel : $\bar{4}2 \to 3\bar{1}$



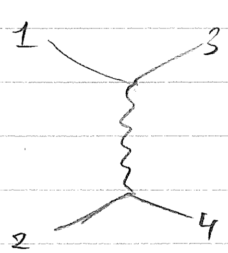
$\theta_{3\bar{4}} \equiv$ angular variable in u-channel
$\equiv s_{34} = s_{12}$ energy variable in s-channel.

Take $\theta_{3\bar{4}} \longrightarrow \infty$ for Regge limit

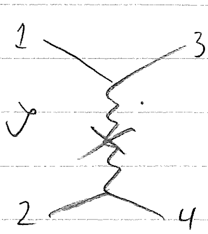
The two contributions have a phase difference of $(-1)^l \equiv e^{i\pi l}$

bad $l \to i\infty$ behavior.

$\Rightarrow$ introduce signature



$+\nu$



More on the Regge signature

The origin of signature is as follows —
it comes ultimately from spin-statistics & indistinguishability — leads to exchange

Signature comes from left-hand cut in $A(t,u)$ sc. ampl.

Left-hand cut comes from exchange graphs
due to indistinguishability of particles.

In potential scattering, left-hand cut is absent.

In continuing the partial wave amplitudes to non-integer ℓ, the question of uniqueness arises.

important because different continuations, would lead to different Regge theories".

$$\underset{\substack{\uparrow \\ \text{different} \\ \text{continuation}}}{A_\ell^{new}(s)} = \underset{\substack{\uparrow \\ \text{chosen} \\ \text{continuation}}}{A_\ell(s)} + \underset{\substack{\uparrow \\ \text{difference} \\ \text{function}}}{f(\ell)}$$

Here, Carlson's theorem is needed.

Let $f(\ell)$ satisfy

(i) Regular in $Re(\ell) > L$ (no singularity past L)

(ii) $f(\ell) < e^{\pi|\ell|}$ (exponential bounded)
for $Re(\ell) > L$

(iii) $f(\ell) = 0$ for an infinite sequence (in our case $\ell \in \mathbb{Z}$)

then $f(\ell) = 0$ for all ℓ,

Therefore, we need a continuation satisfying

① Exponential boundedness. → leads to introducing 'Signature'

① Regularity for $Re(\ell) > L$ → postulate of 'Maximal analyticity of the second kind'