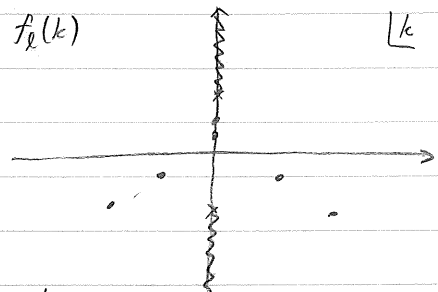
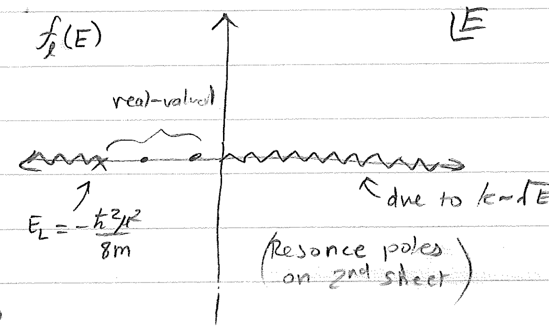
signatured

Singularities of partial-wave amplitudes and dispersion relations

Recall non-relativistic theory:
(Yukawa potential)

$$f_\ell(k) = \frac{1}{2ik}\,\frac{f_\ell(-k) - f_\ell(k)}{f_\ell(k)}$$





How does this look in relativistic theory?

In relativistic theory, consider signaured partial wave amplitudes, ℓ integer.

definition:

$$A_\ell^{\mathcal{S}}(s) = \underset{\substack{\uparrow\\ \text{analytic}\\ \text{in } s}}{\begin{pmatrix}\text{t-channel}\\ \text{pole terms}\end{pmatrix}} + \frac{1}{\pi}\int_{z(t_{th})}^{\infty} dz'\, D_t(s,z')\,Q_\ell(z') + \frac{\mathcal{S}}{\pi}\int_{-z(u_{th})}^{\infty} dz'\, D_u(s,u(-z'))\,Q_\ell(z')$$

for simplicity, assume equal-mass kinematics

$$= \ldots + \frac{1}{\pi}\int_{z(t_{th})}^{\infty} dz'\, \underbrace{\left(D_t(s,z') + \mathcal{S}\, D_u(s,u(-z'))\right)}_{\equiv\, D_t^{\mathcal{S}}(s,z')} Q_\ell(z')$$

$$= \frac{1}{\pi}\int_{z(t_{th})}^{\infty} dz'\, D_t^{\mathcal{S}}(s,z')\,Q_\ell(z') \qquad (*)$$

and also

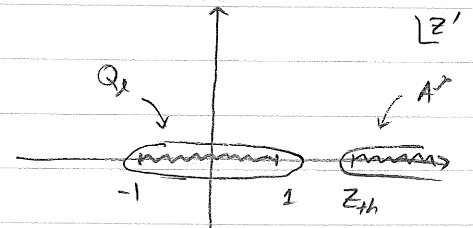
$$A_\ell^{\mathcal{S}}(s) = \frac{1}{2}\int_{-1}^{1} dz'\, A^{\mathcal{S}}(s,t(z'))\,P_\ell(z') \qquad (**)$$

Astute observation: $2D_t^{\mathcal{S}}(s,z')$ is the discontinuity of $A^{\mathcal{S}}(s,t(z'))$ in t

$-\pi P_\ell(z')$ is the discontinuity of $Q_\ell(z')$

So, one can write (combining the two forms)

$$A_\ell^{\nu}(s) = \frac{1}{2\pi}\oint dz'\, A^{\nu}(s, t(z'))\, Q_\ell(z')$$



integration is over either of the two contours.

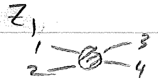
Integration in (**) is over a finite t-region, at fixed s.

$\Rightarrow$ all s-channel branch points of $A^{\nu}(s,t)$ will also appear in $A_\ell^{\nu}(s)$.

These s-channel branch points appear in $D_\ell^{\nu}(s, z')$

But additional singularities in $A_\ell^{\nu}(s)$ will appear due to the partial wave projection.

– arise from vanishing of three-momenta which apper in expression for z, at the threshold in initial state



$$t = \frac{-1}{2s}\lambda(m_1^2, m_2^2, s)(1-z) \equiv -2|\vec{p}_f|^2(1-z) \qquad |\vec{p}_f|^2 = 0$$

$$z = \frac{2st}{\lambda(m_1^2, m_2^2, s)} + 1 \equiv \frac{2t}{|\vec{p}_f|^2} + 1$$

$$0 = |\vec{p}|^2 = \frac{1}{4s}(m_1^2 - m_2^2 + s)^2 - m_1^2$$

$$4 s m_1^2 = (m_1^2 - m_2^2 + s)^2 \qquad s_{th} = (m_1 \pm m_2)^2$$

check: $4(m_1 \pm m_2)^2 m_1^2 = (m_1^2 - \cancel{m_2^2} + m_1^2 + \cancel{m_2^2} \pm 2m_1 m_2)^2$

$= (2m_1^2 \pm 2m_1 m_2)^2$

$= 4m_1^2(m_1 \pm m_2)^2$ ✓

$+ \equiv$ real threshold

$- \equiv$ pseudo threshold.

$$\int \frac{dr}{x} f(s, \tfrac{r}{x})$$