Analytic properties of trajectory functions

– Collins § $\cancel{2.8,}$ 3.2   (and § 5.4 for Hadrons)

$$A^{s}_{Regge}(s,t) = -16\pi^2\,(2\alpha(t)+1)\,\beta(t)\,\frac{P_{\alpha(t)}(-z_t)}{\sin\pi\alpha(t)} \qquad (*)$$

How does discontinuity behave?

<u>Use</u> $\quad \mathrm{Im}\,P_\ell(z) = -P_\ell(-z)\sin(\pi\ell) \quad$ for $z<-1$ $\quad$ <u>$\ell\in\mathbb{R}$.</u>

$\qquad = 0 \quad$ for $z>0$

Then insert into

$$\rho_s(s,t)_{Regge} = 16\pi^2(2\alpha(t)+1)\,\beta(t)\,P_{\alpha(t)}(z_t)$$

right-hand cut

Write fixed t dispersion relation for A^{s}_{Regge}.

inside physical region for s-channel process: $\quad$ in particular, $t<0$; $\ell\in\mathbb{Z}_+$ (incl. zero)

(no pole contrib)

Only $P_{\alpha(t)}(-z_t(s))$ has a discontinuity. Because

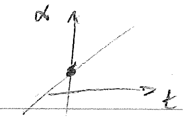
$$A^{s}_{Regge}(s,t) = -16\pi^2(2\alpha(t)+1)\,\beta(t)\,\frac{1}{\sin\pi\alpha(t)}\,\frac{1}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'-s}\,\mathrm{Im}\,P_{\alpha(t)}(z_t(s'))$$

ch. var: $\quad dz = \dfrac{ds}{4|\vec p_{13}||\vec p_{24}|} +$

What is the discontinuity across the s-cut? $\rightarrow$ all comes from $P_{\alpha(t)}$.

$$\mathrm{Disc}\,A^{s}_{Regge}(s,t) = -16\pi^2(2\alpha(t)+1)\,\beta(t)\,\frac{1}{\cancel{\sin\pi\alpha(t)}}\,(-)P_{\alpha(t)}(-z_t)\,\cancel{\sin\pi\alpha(t)}$$

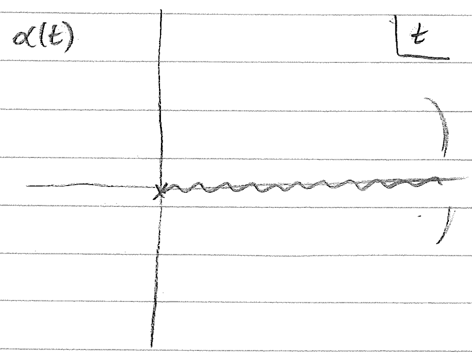
$$= 16\pi^2(2\alpha(t)+1)\,\beta(t)\,P_{\alpha(t)}(-z_t)$$

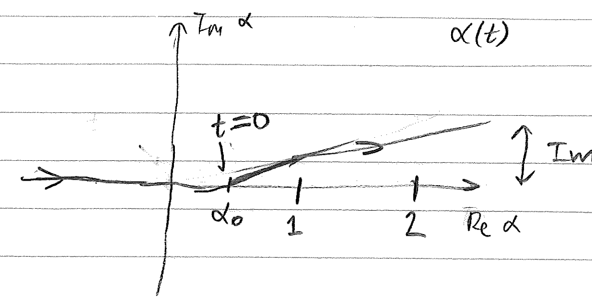


Mandelstam's reasoning for linear trajectories

"The fact that Regge trajectories are almost linear is correlated with their observed narrow widths."

Consider the dispersion relation for Regge trajectory function for a single given pole.





asume $|\alpha(t)| \longrightarrow t$ (need two subtractions)

So write a dispersion relation for $\dfrac{\alpha(t)}{(t-t_1)(t-t_2)}$.

For convenience, subtract at same point: $t_1 = t_2 \equiv t_0 \in \mathbb{R}$ (probably below th.)

Then:

$$\alpha(t) = \alpha_0 + \alpha' t + \frac{(t-t_0)^2}{\pi}\int_0^\infty dt' \, \frac{2i \, \mathrm{Im}\,\alpha(t')}{(t'-t)(t'-t_0)^2}$$

$\mathrm{Im}\,\alpha(t) \sim t$

If $|\alpha(t)| \longrightarrow$ const, then

$$A(s,t) = \alpha_0 + \frac{(t-t_0)}{\pi} \neq \frac{1}{t}$$