One body phase space

$$d(\text{LIPS}) = \frac{d^4p}{(2\pi)^4}(2\pi)\,\delta_+(p^2-m^2)\,\overbrace{(2\pi)^4\,\delta^{(4)}\left(\textstyle\sum p_{in}-p\right)}^{\text{arises from evolution of } \hat{T}.}$$

Integrate over $p^{\mu=0}$

$$d(\text{LIPS}) = \frac{d^3p}{(2\pi)^3}\,\frac{1}{2E}\,(2\pi)^4\,\delta^{(4)}\left(\textstyle\sum p_{in}-p\right)$$

①

$$\delta(E_{in}-E) \equiv \delta\left(\sqrt{p_{in}^2+m^2}-\sqrt{p^2+m^2}\right) \qquad d^3p \equiv dp\,p^2\,d\Omega$$

$$= \sum \frac{1}{\left|\frac{\partial(\,)}{\partial p}\right|_{root}}\,\delta\left(f(p)_{root}\right)$$

$$= \frac{1}{\frac{2p}{2\sqrt{p^2+m^2}}\Big|_{root}}\,\delta(p_{in}-p) + \frac{1}{\frac{2p}{2\sqrt{p^2-m^2}}}\,\delta(p_{in}+p)$$

$$= \frac{\sqrt{p_{in}^2+m^2}}{p_{in}}\left(\delta(p_{in}-p) + \delta(p_{in}+p)\right)$$

this delta will not contrib because integration range is restricted to $0\to+\infty$ only.

②

$$\delta(E_{in}-E) = \frac{1}{\left|\frac{\partial(\,)}{\partial p^2}\right|_{root}}\,\delta\left(f(p^2)_{root}\right)$$

$$= \frac{1}{1/2\sqrt{p^2-m^2}\,\Big|_{root}}\,\delta(p_{in}^2-p^2) \qquad d^3p = \frac{|p|}{2}\left(d|p|^2\right)d\Omega$$

$$= 2\sqrt{p_{in}^2+m^2}\;\delta(p_{in}^2-p^2) \qquad \underline{\text{Ok}}$$