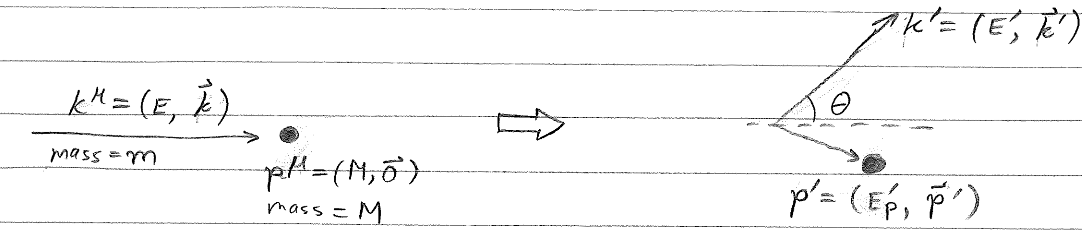
$2 \to 2$ kinematics in LAB frame



$$k \cdot k' = EE' - k \cdot k'$$
$$= EE' - |\vec{k}||\vec{k}'|\cos\theta \longrightarrow EE'(1-\cos\theta)$$
$$= 2EE'\sin^2(\theta/2)$$

--- if $m=0$,

$$k \cdot p = EM$$
$$k' \cdot p = E'M$$

alternative forms can be obtained using $s+t+u = 2m^2 + 2M^2$

$$s = (k+p)^2 = k^2 + p^2 + 2k \cdot p$$
$$= m^2 + M^2 + 2EM$$

$$t = (k-k')^2 = 2m^2 - 2k \cdot k'$$
$$= 2m^2 - 2(EE' - |\vec{k}||\vec{k}'|\cos\theta)$$

if $m=0$: simplifies

$$= -4EE'\sin^2(\theta/2)$$

$$t = -s - u + 2m^2 + 2M^2$$
$$= -(m^2+M^2+2EM) - (m^2+M^2-2E'M) + 2m^2 + 2M^2$$
$$= -2M(E-E')$$

$$u = (k'-p)^2 = (k')^2 + p^2 - 2k' \cdot p$$
$$= m^2 + M^2 - 2E'M$$

Conservation of energy:

relates E' to (E & θ),

set two forms of t equal:

$$\cancel{2}m^2 - \cancel{2}(EE' - |\vec{k}||\vec{k}'|\cos\theta) = -\cancel{2}M(E-E')$$

very complicated for arbitrary m & M

Special cases:

$\boxed{m^2 = 0}$

$$\Rightarrow \quad |\vec{k}| = E \quad \& \quad |\vec{k}'| = E'$$

$$-EE'\underbrace{(1-\cos\theta)}_{2\sin^2(\theta/2)} = -M(E-E')$$

$$2EE'\sin^2(\theta/2) = ME - ME'$$

$$E'(M + 2E\sin^2(\theta/2)) = ME$$

$$E' = \frac{ME}{M + 2E\sin^2\theta/2}$$

$$\boxed{E' = E\,\frac{1}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}}$$

← tells us how recoiling affects E' & E.

$\boxed{M = m}$

$$m^2 - EE' + |\vec{k}||\vec{k}'|\cos\theta = -mE + mE'$$

$$\underbrace{m^2 - EE' + mE - mE'} + \sqrt{E^2-m^2}\sqrt{E'^2-m^2}\cos\theta = 0$$

$$-(E+m)(E'-m) = -\sqrt{E^2-m^2}\sqrt{E'^2-m^2}\cos\theta$$

$$(E+m)^{\cancel{2}}(E'-m)^{\cancel{2}} = (E^2-m^2)(E'^2-m^2)\cos^2\theta$$

$$= (E-m)\cancel{(E+m)}\cancel{(E'-m)}(E'+m)\cos^2\theta$$

$$(E+m)E' - (E+m)m = (E-m)E'\cos^2\theta + (E-m)m\cos^2\theta$$

$$[(E+m) - (E-m)\cos^2\theta]E' = (E+m)m + (E-m)m\cos^2\theta$$

$$\boxed{E' = m\,\frac{E+m+(E-m)\cos^2\theta}{E+m-(E-m)\cos^2\theta}}$$