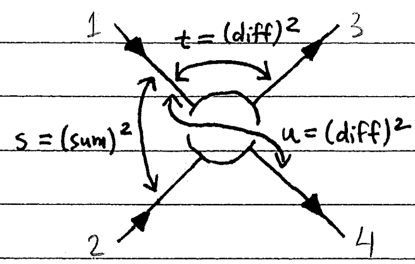
Crossing Symmetry – consequence of (CPT) and Analyticity

If we've calculated the matrix element for a particular process, then we can obtain matrix elements for other processes related by crossing symmetry.

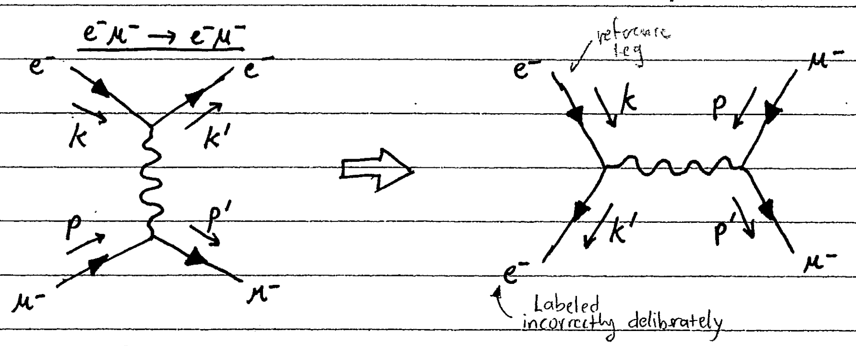
1. Draw the 2 body → 2 body scattering diagram:



– Let arrows indicate flow of momentum, but don't label them, yet.

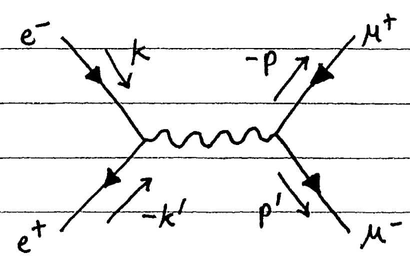
– Indicate and define external momenta-relating Mandelstam variables

2. Draw the Feynman diagram with labeled external momenta associated with calculated matrix element... ... and transform it (holding refrence leg in place) to obtain Feynmam diagram associated with the desired process:



Arrows indicate flow of charge.

In the transformed diagram, some external momenta are pointing in the wrong direction; reverse them with minus signs. But this reverses the sign of energy, too, making them negative. Since we've agreed to call particles with negative ~~energy~~ frequency as antiparticles with positive energy, allow minus signs to act only on the three spatial components of the momenta, but relabel these legs as antiparticles, with their energy left unaffected. – there is a minor subtlety with crossing fermions.



~~Arrows Indicate~~ flow of charge

$e^-e^+ \to \mu^-\mu^+$

3. Compare both Feynman diagrams with the scattering diagram to write down expressions for the Mandelstam variables for both processes:

$e^-\mu^- \longrightarrow e^-\mu^-$	$e^-e^+ \longrightarrow \mu^-\mu^+$	$*$ = for new diagram
$s = (k+p)^2$	$s^* = (k-k')^2$	
$t = (k-k')^2$	$t^* = (k+p)^2$	
$u = (k-p')^2$	$u^* = (k-p')^2$	

Hence, the two processes are related by $s \leftrightarrow t$ and u unchanged.

Thus, if for $e^-\mu^- \rightarrow e^-\mu^-$, $|\overline{\mathcal{M}}|^2 = 2e^4 \dfrac{s^2+u^2}{t^2}$

then, for $e^-e^+ \rightarrow \mu^-\mu^+$, $|\overline{\mathcal{M}}|^2 = 2e^4 \dfrac{t^2+u^2}{s^2}$