

Standard Model - Field content

All fermions in the $(\frac{1}{2}, 0)$ representation of Lorentz group. (Left-handed)

Left-handed quarks $q_f^i(x) = \begin{pmatrix} u^a \\ d^a \end{pmatrix}_f$ $(3, 2, \frac{+1}{6})$

Annotations: SU(2) index points to i , SU(3) index points to a .

using q to avoid confusion with Q , the electric charge operator.

Right-handed up-type quarks $\bar{u}_f^a(x)$ $(\bar{3}, 1, -2/3)$

"Barred" fields are not any sort of conjugate.

Right-handed down-type quarks $\bar{d}_f^a(x)$ $(\bar{3}, 1, 1/3)$

They serve to remind us that fields are in the conjugate representation of SU(3) (and U(1))

Left-handed lepton $l_f^i(x) = \begin{pmatrix} \nu \\ e \end{pmatrix}_f$ $(1, 2, -1/2)$

Right-handed charged lepton $\bar{e}_f(x)$ $(1, 1, +1)$

Scalar fields:

Higgs doublet: $H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$ $(1, 2, 1/2)$
(Scalar)

Gauge bosons:

U(1) gauge field $B_\mu(x)$ $(1, 1, 0)$

SU(2)_L gauge field $W_\mu^a(x)$ $(1, 3, 0)$

SU(3) gauge field $G_\mu^A(x)$ $(8, 1, 0)$
(gluons)

Generators:

$(T_{SU(2)}^A)^a_b = \frac{\lambda^A}{2}$ ← Gell-Mann

$(T_{SU(2)}^a)^i_j = \frac{\tau^a}{2}$ ← Pauli- σ

Y

Lagrangian of the Standard Model of electroweak interactions

Conveniently divided into four parts:

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{Maxwell Yang-Mills}} + \mathcal{L}_{\text{Dirac-Weyl}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

$$\mathcal{L}_{\text{Maxwell Yang-Mills}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{\mu\nu a} - \frac{1}{4} G_{\mu\nu}^A G^{\mu\nu A}$$

where

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad \text{Hypercharge}$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g e^{abc} W_\mu^b W_\nu^c \quad \text{Weak isospin}$$

$$G_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G_\mu^A - g_s f^{ABC} G_\mu^B G_\nu^C \quad \text{colored gluons.}$$

$$\mathcal{L}_{\text{Dirac-Weyl}} = \sum_{\text{generations}} \left[q^\dagger i \bar{\sigma}^\mu D_\mu q + \bar{u}^\dagger i \bar{\sigma}^\mu D_\mu \bar{u} + \bar{d}^\dagger i \bar{\sigma}^\mu D_\mu \bar{d} + l^\dagger i \bar{\sigma}^\mu D_\mu l + \bar{e}^\dagger i \bar{\sigma}^\mu D_\mu \bar{e} \right]$$

where

$$D_\mu q = \left[\partial_\mu + i g_s T_{SU(3)}^A G_\mu^A + i g T_{SU(2)}^a W_\mu^a + i g' \left(\frac{1}{6}\right) B_\mu \right] q$$

$$D_\mu \bar{u} = \left[\partial_\mu + i g_s \bar{T}_{SU(3)}^A G_\mu^A + i g' \left(-\frac{2}{3}\right) B_\mu \right] \bar{u}$$

$$D_\mu \bar{d} = \left[\partial_\mu + i g_s \bar{T}_{SU(3)}^A G_\mu^A + i g' \left(\frac{1}{3}\right) B_\mu \right] \bar{d}$$

$$D_\mu l = \left[\partial_\mu + i g T_{SU(2)}^a W_\mu^a + i g' \left(-\frac{1}{2}\right) B_\mu \right] l$$

$$D_\mu \bar{e} = \left[\partial_\mu + i g' (+1) B_\mu \right] \bar{e}$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D^\mu H) - \underbrace{\left(-\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \right)}_{V(H)}$$

where $D_\mu H = \left[\partial_\mu + i g T^a W_\mu^a + i g' \left(\frac{1}{2}\right) B_\mu \right] H$

$$\mathcal{L}_{\text{Yukawa}} = -\bar{H}_0 q_f^\dagger (\gamma_u)_{fg} \bar{u}_g^\dagger - H_0 q_f^\dagger (\gamma_d)_{fg} \bar{d}_g^\dagger - H_0 l_f^\dagger (\gamma_l)_{fg} \bar{e}_g^\dagger + \text{h.c.}$$

$$\bar{H} = -\epsilon_{ij} H_j^\dagger$$

sum (horizontally) over generations

$(\gamma_u)_{fg}, (\gamma_d)_{fg}, (\gamma_l)_{fg}$ are neither Hermitian nor unitary.