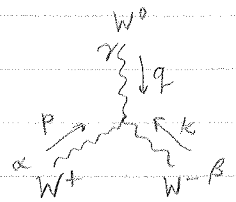
Feynman rule for WWW vertex.



$$\sim -ig\Big(\overset{①}{\langle 0|} (\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) W^{-\mu} W^{3\nu} \,|W^+_\alpha(p) W^-_\beta(k) W^3_\gamma(q)\rangle$$

$$- \overset{②}{\langle 0|} (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) W^+ W^{3\nu} \,|W^+_\alpha(p) W^-_\beta(k) W^3_\gamma(q)\rangle$$

$$+ \overset{③}{\langle 0|} (\partial_\mu W^3_\nu - \partial_\nu W^3_\mu) W^{+\mu} W^{-\nu} \,|W^+_\alpha(p) W^-_\beta(k) W^3_\gamma(q)\rangle$$

Use: $W^+_\mu |W^+_\alpha(p)\rangle \longrightarrow g_{\mu\alpha}$

$\partial_\mu W^+_\nu |W_\alpha(p)\rangle \longrightarrow -ip_\mu g_{\nu\alpha}$

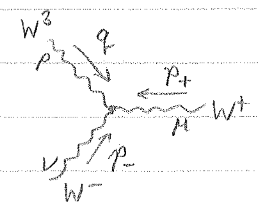
$$① = \underset{\text{Feynman rule}}{i} \times -ig\,(-ip_\mu g_{\nu\alpha} - -ip_\nu g_{\mu\alpha})\,\delta^\mu{}_\beta \delta^\nu{}_\gamma = -ig\,(p_\beta g_{\gamma\alpha} - p_\gamma g_{\beta\alpha})$$

$$② = i \times -ig \times (-1)\,(-ik_\mu g_{\nu\beta} - -ik_\nu g_{\mu\beta})\,\delta^\mu{}_\alpha \delta^\nu{}_\gamma = ig\,(k_\alpha g_{\gamma\beta} - k_\gamma g_{\alpha\beta})$$

$$③ = i \times -ig \times (-iq_\mu g_{\nu\gamma} - -iq_\nu g_{\mu\gamma})\,\delta^\mu{}_\alpha \delta^\nu{}_\beta = -ig\,(q_\alpha g_{\beta\gamma} - q_\beta g_{\alpha\gamma})$$

$$\text{Feynman rule} = -ig\,\big(g_{\alpha\beta}(k-p)_\gamma + g_{\beta\gamma}(q-k)_\alpha + g_{\gamma\alpha}(p-q)_\beta\big)$$

Clean up: $\boxed{SU(2)}$



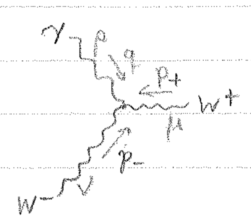
$$= -ig\left(g^{\mu\nu}(p_- - p_+)^\rho + g^{\nu\rho}(q - p_-)^\mu + g^{\rho\mu}(p_+ - q)^\nu\right)$$

combinatoric factor = 1

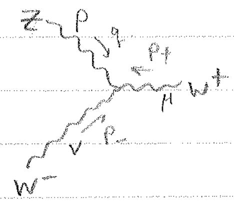
For SM, use $W^3_\mu = \cos\theta_W Z_\mu + \sin\theta_W A_\mu$.

$\boxed{SU(2)\times U(1)}$

agrees with Cheng & Li



$$= -\underset{}{\overset{e = g\sin\theta_W}{ie}}\left(g^{\mu\nu}(p_- - p_+)^\rho + g^{\nu\rho}(q - p_-)^\mu + g^{\rho\mu}(p_+ - q)^\nu\right)$$

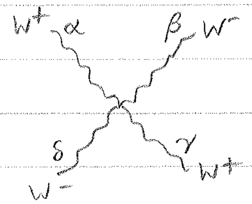


$$= -ig\cos\theta_W\left(g^{\mu\nu}(p_- - p_+)^\rho + g^{\nu\rho}(q - p_-)^\mu + g^{\rho\mu}(p_+ - q)^\nu\right)$$

Feynman rule for $W^+W^-W^+W^-$ vertex

$$\mathcal{L} = \ldots \frac{1}{4}\underbrace{g^{\mu\nu}g^{\nu\sigma}}_{\text{antisymmetrize}\to 1/2}\underbrace{(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)}_{\text{drop 2nd term}\to 2}\underbrace{(W^+_\rho W^-_\sigma - W^+_\sigma W^-_\rho)}_{\text{drop 2nd term}\to 2}$$

$$= \ldots \frac{1}{4}\left[\frac{1}{2}(g^{\mu\nu}g^{\nu\sigma} - g^{\nu\rho}g^{\mu\sigma})\, 2 W^+_\mu W^-_\nu W^+_\rho W^-_\sigma\right]$$



$$\sim \frac{1}{2}g^2(g^{\mu\rho}g^{\nu\sigma} - g^{\nu\rho}g^{\mu\sigma})\langle 0|\, W^+_\mu W^-_\nu W^+_\rho W^-_\sigma \quad W^-_\alpha W^+_\beta W^-_\gamma W^+_\delta |0\rangle$$

There are $2\times 2 = 4$ possible contractions

$$\text{Feynman rule} = i \times \frac{1}{2}g^2(g^{\mu\rho}g^{\nu\sigma} - g^{\nu\rho}g^{\mu\sigma})\times(g_{\alpha\mu}g_{\gamma\rho} + g_{\alpha\rho}g_{\gamma\mu})(g_{\beta\nu}g_{\delta\sigma} + g_{\beta\sigma}g_{\delta\nu})$$

$$= \frac{i}{2}g^2\left[(g_{\alpha\gamma}+g_{\alpha\gamma})(g_{\beta\delta}+g_{\beta\delta}) - (\delta_\alpha{}^\sigma\delta_\gamma{}^\nu + \delta_\alpha{}^\nu\delta_\gamma{}^\sigma)(g_{\beta\nu}g_{\delta\sigma} + g_{\beta\sigma}g_{\delta\nu})\right]$$

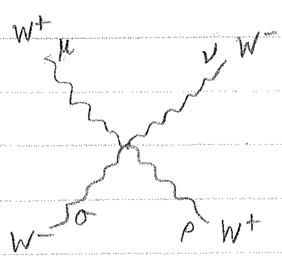
$$= \frac{i}{2}g^2\left[4 g_{\alpha\gamma}g_{\beta\delta} - (\overset{\text{Firsts}}{g_{\beta\gamma}g_{\delta\alpha}} + \overset{\text{outers}}{g_{\beta\alpha}g_{\delta\gamma}} + \overset{\text{inners}}{g_{\beta\alpha}g_{\delta\gamma}} + \overset{\text{lasts}}{g_{\beta\gamma}g_{\delta\alpha}})\right]$$

$$= \frac{i}{2}g^2\left[4 g_{\alpha\gamma}g_{\beta\delta} - 2g_{\alpha\beta}g_{\gamma\delta} - 2g_{\alpha\delta}g_{\beta\gamma}\right]$$

$$= -ig^2(g_{\alpha\beta}g_{\gamma\delta} - 2g_{\alpha\gamma}g_{\beta\delta} + g_{\alpha\delta}g_{\beta\gamma})$$

Clean up

agrees with Cheng & Li



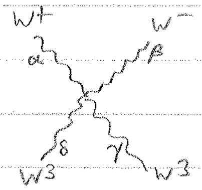
$$= -ig^2(\overset{\text{opposite charge}}{g^{\mu\nu}g^{\rho\sigma}} - \underset{\text{like-charge}}{2g^{\mu\rho}g^{\nu\sigma}} + \overset{\text{opposite charge}}{g^{\mu\sigma}g^{\nu\rho}})$$

combinatoric factor $= \frac{1}{4}$

Feynman rule for $W^+W^-W^3W^3$

$$\mathcal{L} = \cdots \underbrace{- \frac{1}{2} g^{\mu\rho} g^{\nu\sigma}}_{\text{antisymmetrize} \to 1/2} \underbrace{(W^+_\mu W^3_\nu - W^+_\nu W^3_\mu)}_{\text{drop 2nd term} \to 2} \underbrace{(W^-_\rho W^3_\sigma - W^-_\sigma W^3_\rho)}_{\text{drop 2nd term} \to 2}$$

$$= \cdots - \frac{1}{2}\left[\frac{1}{2}(g^{\mu\rho}g^{\nu\sigma} - g^{\nu\rho}g^{\mu\sigma})\, 2W^+_\mu W^3_\nu \; 2W^-_\rho W^3_\sigma\right]$$



$$\sim -g^2(g^{\mu\rho}g^{\nu\sigma} - g^{\nu\rho}g^{\mu\sigma}) \langle 0| W^+_\mu W^3_\nu \; W^-_\rho W^3_\sigma \; W^-_\alpha W^+_\beta W^3_\gamma W^3_\delta |0\rangle$$

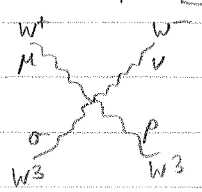
There are 2 possible contractions ($W^3W^3 = A,A$ or Z,Z)
(There is 1 possible contraction if $W^3W^3 = AZ$, but extra ZA cross term)

Feynman rule $= -ig^2(g^{\mu\rho}g^{\nu\sigma} - g^{\nu\rho}g^{\mu\sigma})\, g_{\mu\alpha}g_{\rho\beta}(g_{\gamma\nu}g_{\delta\sigma} + g_{\gamma\sigma}g_{\delta\nu})$

$$= -ig^2\left(g_{\alpha\beta}(g_{\gamma\delta} + g_{\gamma\delta}) - \delta^\sigma{}_\alpha \delta^\nu{}_\beta (g_{\gamma\nu}g_{\delta\sigma} + g_{\gamma\sigma}g_{\delta\nu})\right)$$

$$= ig^2\left(g_{\alpha\gamma}g_{\beta\delta} - 2g_{\alpha\beta}g_{\gamma\delta} + g_{\beta\gamma}g_{\alpha\delta}\right)$$

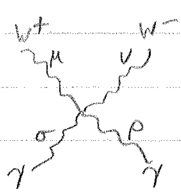
clean up: $\boxed{SU(2)}$



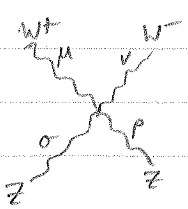
$$= ig^2(g^{\mu\rho}g^{\nu\sigma} - 2g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho})$$

combinatoric factor $= \frac{1}{2}$

For SM, $W^3_\mu = \cos\theta_W Z_\mu + \sin\theta_W A_\mu$ $\boxed{SU(2)\times U(1)}$

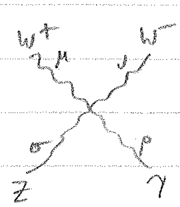


$$= ie^2(g^{\mu\rho}g^{\nu\sigma} - 2g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho})$$



$$= ig^2\cos^2\theta_W(g^{\mu\rho}g^{\nu\sigma} - 2g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho})$$

combinatoric factor $= \frac{1}{2}$

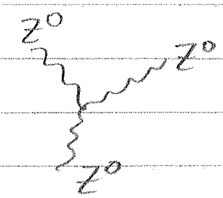


$$= ieg\cos\theta_W(g^{\mu\rho}g^{\nu\sigma} - 2g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho})$$

combinatoric factor $= 1$

<u>Notice</u> — there are no tree-level vertices of the type:

$$\underbrace{ZZZ}_{\substack{\uparrow \\ \text{To understand} \\ \text{this, write:}}}, \quad \underbrace{\gamma ZZ, \quad \gamma\gamma Z \quad \text{or} \quad \gamma\gamma\gamma}_{\substack{\text{this is obvious: } \gamma \text{ couples} \\ \text{to charged fields.}}}$$



absent!

$$Z = \cos\theta \, W - \sin\theta \, B$$

$$\Rightarrow \quad ZZZ \simeq \underbrace{W^3W^3W^3}_{} + \underbrace{W^3W^3B + W^3BB + BBB}_{\substack{\text{these are not present because } B \\ \text{couples to hyper-charged fields.} \\ (W^3 \text{ has no hyper charge, } Y=0)}}$$

FINALLY: this is zero for elementary group-theoretic reasons:

Non-abelian gauge self couplings always involve the antisymmetric structure constants $f^{abc} = \epsilon^{abc}$ for $SU(2)$.

$$\Rightarrow \quad \mathcal{L} = \ldots g \, \epsilon^{abc} \partial_\mu W^{\mu a} W_\nu^b W^{\nu c}$$

will <u>never contain</u> $W^3W^3W^3$.

n.b.: similar arguments for absence of $ZZZZ$ coupling.

$$\mathcal{L} = \ldots \epsilon^{abc} \epsilon^{cde} W^a W^b W^d W^e$$

(ab) nor (de) may be identical.