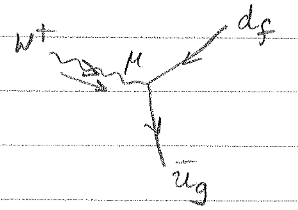
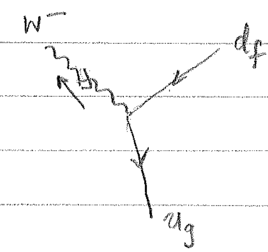
Charged currents

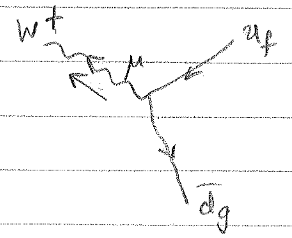
— depends only on flow of charge.

Quarks

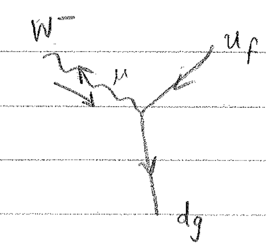


$$= \frac{-ig}{2\sqrt{2}}\left(\gamma^\mu - \gamma^\mu\gamma_5\right)\left(V_{CKM}\right)_{fg} =$$

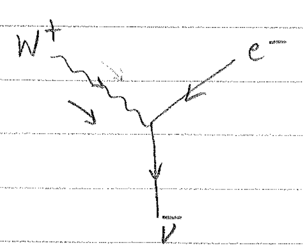




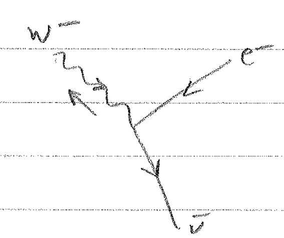
$$= \frac{-ig}{2\sqrt{2}}\left(\gamma^\mu - \gamma^\mu\gamma_5\right)\left(V^*_{CKM}\right)_{gf} =$$

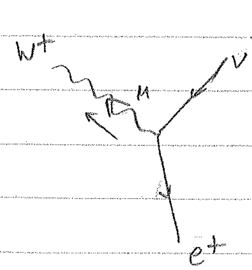


Leptons

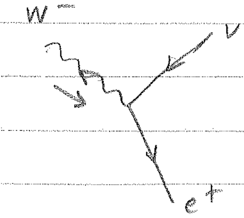


$$= \frac{-ig}{2\sqrt{2}}\left(\gamma^\mu - \gamma^\mu\gamma_5\right) =$$





$$= \frac{-ig}{2\sqrt{2}}\left(\gamma^\mu - \gamma^\mu\gamma_5\right) =$$



Summary

Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix.

Origin:

To diagonalize the two mass matrices Y_u and Y_d.

We need to mix these ...

u	c	t
d′	s′	b′

... and these independently.

But these are paired up in a special way: They form SU(2) doublets.

Definition:

V_{CKM} maps down-type quarks in mass-eigenstate to gauge-eigenstate:

$$\underbrace{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}}_{\text{gauge eigenstate}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}_{\text{mass eigenstates}}$$

Magnitude of CKM elements: (as of 2012)

$$|V_{CKM}| = \begin{pmatrix} 0.9742 & 0.2253 & 0.0035 \\ 0.2252 & 0.9734 & 0.041 \\ 0.00867 & 0.040 & 0.99915 \end{pmatrix}$$

Summary —

Three terms in $\mathcal{L}$:

c.c interaction $(u_1\ u_2\ u_3)^\dagger\, i\bar{\sigma}^\mu \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$ $\quad (u\ c\ t)^\dagger\, i\bar{\sigma}^\mu \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$ $\quad (u\ c\ t)^\dagger\, i\bar{\sigma}^\mu V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

up-type quark mass $(u_1\ u_2\ u_3)^\dagger \left(Y_u^{raw} \right) \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{pmatrix}^\dagger \overset{①②}{\Longrightarrow} (u\ c\ t)^\dagger \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{c} \\ \bar{t} \end{pmatrix}^\dagger \overset{③}{\Longrightarrow} (u\ c\ t)^\dagger \begin{pmatrix} \cdot & & \\ & \cdot & \\ & & \cdot \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{c} \\ \bar{t} \end{pmatrix}^\dagger$

down-type quark mass $(d_1\ d_2\ d_3)^\dagger \left(Y_d^{raw} \right) \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \end{pmatrix}^\dagger$ $\quad (d'\ s'\ b')^\dagger \left(Y_d \right) \begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix}^\dagger$ $\quad (d\ s\ b)^\dagger \begin{pmatrix} \cdot & & \\ & \cdot & \\ & & \cdot \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}^\dagger$

① Make right-handed $U(3)^u_R$ and $U(3)^d_R$ rotations to render Y_u & Y_d Hermitian.

$$\begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{pmatrix} = U_R^{(u)} \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{pmatrix} \qquad \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \end{pmatrix} = U_R^{(d)} \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \end{pmatrix}$$

② Make a $U(3)^u_V$ rotation to diagonalize Y_u

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = V_u^\dagger \begin{pmatrix} u \\ c \\ t \end{pmatrix} \qquad \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{pmatrix} = V_u \begin{pmatrix} \bar{u} \\ \bar{c} \\ \bar{t} \end{pmatrix}$$

To preserve $SU(2)$ structure, do the same rotation in d quark sector.

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = V_d^\dagger \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} \qquad \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \end{pmatrix} = V_d \begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix}$$

③ Make an $SU(2)$ misaligning transformation to diagonalize Y_d

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad \begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix} = V_{CKM}^\dagger \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}$$