Custodial Isospin Symmetry – Euclidean $SO(4)$

Higgs doublet has four real degrees of freedom – can be arranged in a column-vector:

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \longrightarrow \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}$$

with $SU(2)\times U(1)$ transformation rule: $\phi_i \longrightarrow (e^{i\alpha^a T^a})_{ij}\,\phi_j$

where

$$T^1 = \frac{1}{2}\left(\begin{array}{cc|cc} & & & i \\ & & -i & \\ \hline & i & & \\ -i & & & \end{array}\right),\quad T^2 = \frac{1}{2}\left(\begin{array}{cc|cc} & & -i & \\ & & & -i \\ \hline i & & & \\ & i & & \end{array}\right),\quad T^3 = \frac{1}{2}\left(\begin{array}{cc|cc} & i & & \\ -i & & & \\ \hline & & & -i \\ & & i & \end{array}\right),\quad Y = \left(\begin{array}{cc|cc} & i/2 & & \\ -i/2 & & & \\ \hline & & & i/2 \\ & & -i/2 & \end{array}\right)$$

For now, drop Yukawa couplings and set hypercharge $Y=0$.
Higgs Lagrangian reads:

$$\mathcal{L}_{Higgs} = \frac{1}{2}(D_\mu\phi)(D^\mu\phi) - \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}(\phi^2)^2$$

This is invariant under $SO(4)$ transformations that mix up real components of ϕ

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} \to \left(e^{\frac{i}{2}\alpha^{kl}(T_V^{kl})}\right)\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}.$$

– form inspired by Lorentz group.

Generators of (Euclidean) Lorentz group are known:

$$(T_V^{kl})^{ij} = -i\left(\delta^{ki}\delta^{lj} - \delta^{kj}\delta^{li}\right) = \left(\begin{array}{ccc|c} 0 & J_3 & -J_2 & \\ & 0 & J_1 & \\ & & 0 & \\ \hline K_1 & K_2 & K_3 & \end{array}\right)$$

__Algebra of $SO(4)$:__

$$[T^{ij}, T^{kl}] = i\left(\delta^{ik}T^{jl} - \delta^{il}T^{jk} - \delta^{jk}T^{il} + \delta^{jl}T^{ik}\right)$$

"$SO(3)_V$"

$$J_1 = \left(\begin{array}{ccc|c} 0 & & & \\ & 0 & -i & \\ & i & 0 & \\ \hline & & & 0 \end{array}\right)\quad J_2 = \left(\begin{array}{ccc|c} 0 & & i & \\ & 0 & & \\ -i & & 0 & \\ \hline & & & 0 \end{array}\right)\quad J_3 = \left(\begin{array}{ccc|c} 0 & -i & & \\ i & 0 & & \\ & & 0 & \\ \hline & & & 0 \end{array}\right)$$

"$SO(3)_A$"

$$K_1 = \left(\begin{array}{ccc|c} & & & i \\ & & & 0 \\ & & & 0 \\ \hline -i & 0 & 0 & \end{array}\right)\quad K_2 = \left(\begin{array}{ccc|c} & & & 0 \\ & & & i \\ & & & 0 \\ \hline 0 & -i & 0 & \end{array}\right)\quad K_3 = \left(\begin{array}{ccc|c} & & & 0 \\ & & & 0 \\ & & & i \\ \hline 0 & 0 & -i & \end{array}\right)$$

__Algebra:__

$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

$$[J_i, K_j] = i\epsilon_{ijk}K_k$$

$$[K_i, K_j] = i\epsilon_{ijk}J_k$$

Group theoretically speaking, $SO(4)$ is reducible:

$$SO(4) \simeq SU(2)_{\text{Weak}} \times SU(2)_{cus}$$

[in analogy with L group]

$\boxed{\text{But NOT hermitian conjugates of one another}}$

– one $SU(2)$ is the <u>electroweak isospin</u>

– the other $SU(2)$ is an (approximate) accidental symmetry: <u>Custodial isospin</u>

Generators of weak isospin $SU(2)$

are linear combinations of J_is and k_is.

Easy task:

"$SO(3)_L$"

$$T^1 = \frac{1}{2}\left(\begin{array}{cc|cc} & & & i \\ & & -i & \\ \hline & i & & \\ -i & & & \end{array}\right) \qquad T^2 = \frac{1}{2}\left(\begin{array}{cc|cc} & & -i & \\ & & & -i \\ \hline i & & & \\ & i & & \end{array}\right) \qquad T^3 = \frac{1}{2}\left(\begin{array}{cc|cc} & i & & \\ -i & & & \\ \hline & & & -i \\ & & i & \end{array}\right)$$

$$= \frac{1}{2}(J_1 + k_1) \qquad = -\frac{1}{2}(J_2 + k_2) \qquad = -\frac{1}{2}(J_3 + k_3)$$

(needed to match weak isospin generators)

(c.f. the N & $N^\dagger$ of Lorentz group)

these <u>minus signs</u> are a pain. – just need to be careful

They don't mess up the usual commutation relations:

$$[T^a, T^b] = i\epsilon^{abc}T^c$$

Generators of Custodial isospin $SU(2)$

are the orthogonal combination to T^a's

"$SO(3)_R$"

$$T^1_{cus} = \frac{1}{2}(J_1 - k_1) \qquad T^2_{cus} = \frac{1}{2}(J_2 - k_2) \qquad T^3_{cus} = \frac{1}{2}(J_3 - k_3)$$

$$= \frac{1}{2}\left(\begin{array}{cc|cc} & & & -i \\ & & -i & \\ \hline & i & & \\ i & & & \end{array}\right) \qquad = \frac{1}{2}\left(\begin{array}{cc|cc} & & i & \\ & & & -i \\ \hline -i & & & \\ & i & & \end{array}\right) \qquad = \frac{1}{2}\left(\begin{array}{cc|cc} & -i & & \\ i & & & \\ \hline & & & -i \\ & & i & \end{array}\right)$$

In the $(\phi^+, \phi^-, \phi^0, \phi^{0*})$ basis

$$T^3_{cus} = \frac{1}{2}\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix},$$

custodial transformations implemented:

$$\phi_i = \left(e^{i\alpha^a T^a_{cus}}\right)_{ij}\phi_j$$

hence, the custodial isospin assignments

Electroweak isospin	custodial isospin UP	custodial isospin DOWN
UP	ϕ^{0*}	ϕ^+
DOWN	ϕ^-	ϕ^0

Coupling to Fermions

In the Standard Model, the Yukawa couplings to (one generation of) quarks reads:

$$\mathcal{L}_{Yuk} = -y_u\,(\epsilon_{ij}H_j)\,q_i\,\bar{u} - y_d\,H_i^\dagger\,q_i\,\bar{d} + \text{h.c.}$$

$q \equiv$ quark doublet
$\bar{u} =$ up quark (isospin singlet)
$\bar{d} =$ down quark (isospin singlet)

where
$$\begin{cases} \epsilon_{ij}H_j = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^0 \\ -\phi^+ \end{pmatrix} \\ H^\dagger = \begin{pmatrix} \phi^- \\ \phi^{0*} \end{pmatrix} \end{cases}$$

$$= -y_u(\phi^0 \;\; -\phi^+)\begin{pmatrix} u \\ d \end{pmatrix}\bar{u} - y_d(\phi^- \;\; \phi^{0*})\begin{pmatrix} u \\ d \end{pmatrix}\bar{d} + \text{h.c.}$$

$$= -y_u\left(\phi^0\,u\bar{u} - \phi^+\,d\bar{u}\right) - y_d\left(\phi^-\,u\bar{d} + \phi^{0*}\,d\bar{d}\right) + \text{h.c}$$

Here, $q \equiv$ electroweak isospin doublet $\Rightarrow$ in SO(4) language, must transform like a $(\tfrac{1}{2}, 0)$ spinor

The Yukawa couplings break the SO(4) symmetry, but if $y_u = y_d \equiv y$, then we could arrange u and d into a $(0, \tfrac{1}{2})$ doublet.

Finding generators for $(\tfrac{1}{2}, 0)$ and $(0, \tfrac{1}{2})$ representations

Introduce $\sigma^i_{a\dot{b}} = (\sigma_1, \sigma_2, \sigma_3, i\mathbb{1})$ and $\bar{\sigma}^{i\dot{a}b} = (-\sigma_1, -\sigma_2, -\sigma_3, i\mathbb{1})$

(Euclidean sigma matrices)

where
$$\sum_{i=1}^{4} \sigma^i_{a\dot{b}}\,\sigma^i_{c\dot{d}} = -2\epsilon_{ac}\epsilon_{\dot{b}\dot{d}}, \qquad (\sigma^i\bar{\sigma}^j + \sigma^j\bar{\sigma}^i)_a{}^b = -2\delta^{ij}\delta_a{}^b$$

$$\sum_{i=1}^{4} \bar{\sigma}^{i\dot{a}b}\,\bar{\sigma}^{i\dot{c}d} = -2\epsilon^{\dot{a}\dot{c}}\epsilon^{bd} \qquad (\bar{\sigma}^i\sigma^j + \bar{\sigma}^j\sigma^i)^{\dot{a}}{}_{\dot{b}} = -2\delta^{ij}\delta^{\dot{a}}{}_{\dot{b}}$$

and
$$\epsilon^{ab} = \epsilon^{\dot{a}\dot{b}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Matrix inverses.

$$\epsilon_{ab} = \epsilon_{\dot{a}\dot{b}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Now, if we say $q_a \longrightarrow \left(e^{\frac{i}{2}\alpha^{kl} T_L^{kl}}\right)_a{}^b q_b$ transformation law of $(1/2, 0)$

and $\bar{q}^{\dot a} \longrightarrow \left(e^{\frac{i}{2}\alpha^{kl} T_R^{kl}}\right)^{\dot a}{}_{\dot b} \bar{q}^{\dot b}$, " " " $(0, 1/2)$

can find T_L^{kl} and T_R^{kl} by requiring <u>invariance of $\sigma^i_{a\dot b}$</u> under L. transformations

$$\sigma^i_{a\dot b} \rightarrow \left(\delta^{ij} + \tfrac{i}{2}\alpha^{kl}(T_V^{kl})^{ij}\right)\left(\delta_a{}^c + \tfrac{i}{2}\alpha^{kl}(T_L^{kl})_a{}^c\right)\left(\delta_{\dot b}{}^{\dot d} + \tfrac{i}{2}\alpha^{kl}(T_R^{kl})_{\dot b}{}^{\dot d}\right)\sigma^j_{c\dot d}$$

$$= \sigma^i_{a\dot b} + \tfrac{i}{2}\alpha^{kl}\Big((T_V^{kl})^{ij}\sigma^j_{a\dot b} + (T_L^{kl})_a{}^c\sigma^i_{c\dot b} + (T_R^{kl})_{\dot b}{}^{\dot d}\sigma^i_{a\dot d}\Big) + O(\alpha)^2$$

↑ Stuff in parenthesis must vanish. $(T_V^{kl})^{ij} = i(\delta^{kj}\delta^{li} - \delta^{ki}\delta^{lj})$

$$0 = i\left(\delta^{li}\sigma^k_{a\dot b} - \delta^{ki}\sigma^l_{a\dot b}\right) + (T_L^{kl})_a{}^c\sigma^i_{a\dot b} + (T_R^{kl})_{\dot b}{}^{\dot d}\sigma^i_{a\dot d}$$

Multiply at left by $\sigma^i_{e\dot e}$

$$= i\left(\sigma^l_{e\dot e}\sigma^k_{a\dot b} - \sigma^k_{e\dot e}\sigma^l_{a\dot b}\right) + (T_L^{kl})_a{}^c\underbrace{\sigma^i_{e\dot e}\sigma^i_{c\dot b}}_{-2\epsilon_{ec}\epsilon_{\dot e\dot b}} + (T_R^{kl})_{\dot b}{}^{\dot d}\underbrace{\sigma^i_{e\dot e}\sigma^i_{a\dot d}}_{-2\epsilon_{ea}\epsilon_{\dot e\dot d}}$$

$$= i\left(\sigma^l_{e\dot e}\sigma^k_{a\dot b} - \sigma^k_{e\dot e}\sigma^l_{a\dot b}\right) - 2\epsilon_{\dot e\dot b}\underbrace{(T_L^{kl})_{ae}}_{\text{symm } a\leftrightarrow e} - 2\epsilon_{ea}\underbrace{(T_R^{kl})_{\dot b\dot e}}_{\text{symm } \dot b\leftrightarrow\dot e}$$

Multiply at left by $\epsilon^{\dot b\dot e}$ to kill 3rd term (solving for T_L)

$$0 = i\epsilon^{\dot b\dot e}\left(\sigma^l_{e\dot e}\sigma^k_{a\dot b} - \sigma^k_{e\dot e}\sigma^l_{a\dot b}\right) - 2\underbrace{\epsilon^{\dot b\dot e}\epsilon_{\dot e\dot b}}_{+2}(T_L^{kl})_{ae} + 0$$

Multiply at left by ϵ^{ce} to clean up.

$$\Rightarrow \boxed{(T_L^{kl})_a{}^c = \frac{i}{4}\left(\sigma^k_{a\dot b}\bar\sigma^{l\dot b c} - \sigma^l_{a\dot b}\bar\sigma^{k\dot b c}\right)}$$

similarly

$$\boxed{(T_R^{kl})^{\dot a}{}_{\dot c} = \frac{i}{4}\left(\bar\sigma^{k\dot a b}\sigma^l_{b\dot c} - \bar\sigma^{l\dot a b}\sigma^k_{b\dot c}\right)}$$

– can read off J_i s and K_i s : (Mathematica)

<u>Left-handed rep: $(1/2, 0)$</u>

act on Q_a

$$J_1 = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \quad J_2 = \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix} \quad J_3 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \quad K_1 = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \quad K_2 = \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix} \quad K_3 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

Notice $T^i_{cus} = 1/2\,(J_i - K_i) = 0$ as expected.

<u>Right-handed rep: $(0, 1/2)$</u>

act on $q^{\dot a}$

$$J_1 = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \quad J_2 = \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix} \quad J_3 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \quad K_1 = \begin{pmatrix} 0 & -1/2 \\ -1/2 & 0 \end{pmatrix} \quad K_2 = \begin{pmatrix} 0 & i/2 \\ -i/2 & 0 \end{pmatrix} \quad K_3 = \begin{pmatrix} -1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

and $T^i \sim 1/2\,(J_i + K_i) = 0$ as expected.

<u>Electroweak and custodial isospin assignments of quarks</u>

Slight complication: Recall the minus sign when mapping generators of $SU(2)_{weak}$ to generators of $SO(4)$, in particular:

$$T^3_{weak} = -\frac{1}{2}(J_3 + K_3) \qquad \left[\text{and also } T^2_{weak} = -\frac{1}{2}(J_2 + K_2)\right]$$

In $(1/2, 0)$-rep, $= \begin{pmatrix} -1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$ ← isospin down is 1st component ← isospin up is 2nd component

This means we must arrange <u>quark isospin doublet</u> like: $q_a = \begin{pmatrix} d \\ u \end{pmatrix}$ literally, up-side (is isospin) down!

(similarly, $l_a = \begin{pmatrix} e \\ \nu \end{pmatrix}$, up-side down)

Fortunately, we defined $T^3_{cus} = +1/2\,(J_3 - K_3)$, so that we can define the <u>custodial isospin doublet</u> (actually broken by hypercharge and Yukawa coupl.)

$$\bar{q}^{\dot a} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$$

Yukawa couplings with full SO(4) symmetry:

Invariant combinations are

$$\bar{q}_{\dot{a}}\,[H.\bar{\sigma}]^{\dot{a}b}\,q_b \qquad \text{or} \qquad \underset{\substack{\uparrow \\ \text{choose this one}}}{q^a\,[H.\sigma]_{a\dot{b}}\,\bar{q}^{\dot{b}}}$$

(related by passing spinors though one another)

$$q_a = \begin{pmatrix} d \\ u \end{pmatrix} \quad \Rightarrow \quad q^a = \epsilon^{ab} q_b = \begin{pmatrix} u \\ -d \end{pmatrix}$$

$$\bar{q}^{\dot{a}} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \quad \Rightarrow \quad \bar{q}_{\dot{a}} = \epsilon_{\dot{a}\dot{b}}\bar{q}^{\dot{b}} = \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

$$\mathcal{L} = -\frac{y}{\sqrt{2}}\, Q^a\,[H.\sigma]_{a\dot{b}}\,q^{\dot{b}} + \text{h.c.}$$ (single generation of quarks)

$$= -\frac{y}{\sqrt{2}}\,(u \;\; -d)\begin{pmatrix} \phi_3+i\phi_4 & \phi_1-i\phi_2 \\ \phi_1+i\phi_2 & -\phi_3+i\phi_4 \end{pmatrix}\begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} + \text{h.c.}$$

$$= -y\,(u \;\; -d)\begin{pmatrix} \phi^0 & \phi^- \\ \phi^+ & -\phi^{0*} \end{pmatrix}\begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} + \text{h.c.}$$

$$= -y\left(\phi^0 u\bar{u} + \phi^- u\bar{d} - \phi^+ d\bar{u} + \phi^{0*} d\bar{d}\right) + \text{h.c.}$$

matches SM yukawa couplings, if $y_u = y_d \equiv y$.

■ Euclidean SO(4) generators

(defining representation)

```
δ[x_, y_] = KroneckerDelta[x, y];

T = Table[Table[ⅈ (δ[k, j] δ[l, i] - δ[k, i] δ[l, j]), {k, 1, 4}, {l, 1, 4}], {i, 1, 4}, {j, 1, 4}];
Table[T⟦i, j⟧ // MatrixForm, {i, 1, 4}, {j, 1, 4}] // MatrixForm
```

$$\begin{pmatrix}
\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&-i&0&0\\i&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&-i&0\\0&0&0&0\\i&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&-i\\0&0&0&0\\0&0&0&0\\i&0&0&0\end{pmatrix}\\
\begin{pmatrix}0&i&0&0\\-i&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&-i&0\\0&i&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&-i\\0&0&0&0\\0&i&0&0\end{pmatrix}\\
\begin{pmatrix}0&0&i&0\\0&0&0&0\\-i&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&i&0\\0&-i&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&-i\\0&0&i&0\end{pmatrix}\\
\begin{pmatrix}0&0&0&i\\0&0&0&0\\0&0&0&0\\-i&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&i\\0&0&0&0\\0&-i&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&i\\0&0&-i&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}
\end{pmatrix}$$

works just as well
(no sign change)

■ Spinor representation

```
σ = {(0 1; 1 0), (0 -ⅈ; ⅈ 0), (1 0; 0 -1), (ⅈ 0; 0 ⅈ)};

σb = {-(0 1; 1 0), -(0 -ⅈ; ⅈ 0), -(1 0; 0 -1), (ⅈ 0; 0 ⅈ)};

ϵ = (0 1; -1 0);
```

$$\sigma = \left\{\begin{pmatrix}0&1\\1&0\end{pmatrix}, \begin{pmatrix}0&-i\\i&0\end{pmatrix}, \begin{pmatrix}1&0\\0&-1\end{pmatrix}, \begin{pmatrix}i&0\\0&i\end{pmatrix}\right\};$$

$$\sigma b = \left\{-\begin{pmatrix}0&1\\1&0\end{pmatrix}, -\begin{pmatrix}0&-i\\i&0\end{pmatrix}, -\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \begin{pmatrix}i&0\\0&i\end{pmatrix}\right\};$$

$$\epsilon = \begin{pmatrix}0&1\\-1&0\end{pmatrix};$$

Checking $\sigma^i_{a\dot{b}}\,\sigma^i_{c\dot{d}} = -2\,\epsilon_{ac}\,\epsilon_{\dot{b}\dot{d}}$

```
Table[Table[Sum[σ⟦i, a, b⟧ σ⟦i, c, d⟧, {i, 1, 4}], {a, 1, 2}, {c, 1, 2}] // MatrixForm,
  {b, 1, 2}, {d, 1, 2}] // MatrixForm
```

$$\begin{pmatrix}\begin{pmatrix}0&0\\0&0\end{pmatrix} & \begin{pmatrix}0&-2\\2&0\end{pmatrix}\\ \begin{pmatrix}0&2\\-2&0\end{pmatrix} & \begin{pmatrix}0&0\\0&0\end{pmatrix}\end{pmatrix}$$

Checking $\bar{\sigma}^{i\dot{a}b}\,\bar{\sigma}^{i\dot{c}d} = -2\,\epsilon^{\dot{a}\dot{c}}\,\epsilon^{bd}$

```
Table[Table[Sum[σb⟦i, a, b⟧ σb⟦i, c, d⟧, {i, 1, 4}], {a, 1, 2}, {c, 1, 2}] // MatrixForm,
  {b, 1, 2}, {d, 1, 2}] // MatrixForm
```

$$\begin{pmatrix}\begin{pmatrix}0&0\\0&0\end{pmatrix} & \begin{pmatrix}0&-2\\2&0\end{pmatrix}\\ \begin{pmatrix}0&2\\-2&0\end{pmatrix} & \begin{pmatrix}0&0\\0&0\end{pmatrix}\end{pmatrix}$$