Chiral anomalies of abelian currents in SM

Recall that the global chiral anomaly due to a single Weyl fermion χ_α coupled to (abelian or non-abelian) gauge fields is:

$$\chi_\alpha \to e^{-i\alpha \hat{G}} \chi_\alpha \xrightarrow{\text{Noether}} (\partial \cdot J^\mu_G)_{\text{anom.}} = \frac{+g^2}{(4\pi)^2} \underbrace{\tfrac{1}{2}\mathrm{Tr}\left[G\{T^a, T^b\}\right]} \tilde{F}^a_{\mu\nu} F^{\mu\nu b}$$

Anomaly coefficient $= G \frac{1}{2}\delta^{ab} \dim(T)_{\text{all other groups}}$

$= G \; Y^2 \dim(Y)_{\text{all other groups}}$

[G generates the symmetry transformations associated with Noether current $J^\mu_G = \chi^\dagger \bar{\sigma}^\mu G \chi$]

Abelian chiral currents carried by each fermion multiplet of a given generation f:

TRANSFORMATION		NOETHER CURRENT
$\begin{pmatrix} u \\ d \end{pmatrix}_f \to e^{-i\alpha} \begin{pmatrix} u \\ d \end{pmatrix}_f$	:	$J^\mu_{q_f} = (q^\dagger \bar{\sigma}^\mu q)_f$
$\bar{u}_f \to e^{-i\alpha} \bar{u}_f$	:	$J^\mu_{\bar{u}} = (\bar{u}^\dagger \bar{\sigma}^\mu \bar{u})_f = -\bar{u}\sigma^\mu \bar{u}^\dagger$
$\bar{d}_f \to e^{-i\alpha} \bar{d}_f$	:	$J^\mu_{\bar{d}} = (\bar{d}^\dagger \bar{\sigma}^\mu \bar{d})_f = -\bar{d}\sigma^\mu \bar{d}^\dagger$
$\begin{pmatrix} \nu \\ e \end{pmatrix}_f \to e^{-i\alpha} \begin{pmatrix} \nu \\ e \end{pmatrix}_f$	:	$J^\mu_\ell = (\ell^\dagger \bar{\sigma}^\mu \ell)_f$
$\bar{e}_f \to e^{-i\alpha} \bar{e}_f$	:	$J^\mu_{\bar{e}} = (\bar{e}^\dagger \bar{\sigma}^\mu \bar{e})_f = -\bar{e}\sigma^\mu \bar{e}^\dagger$

Anomalous contribution to divergence of chiral currents (of a given generation f):

$$\partial \cdot J_{q_f} = \frac{g_s^2}{(4\pi)^2}\left(2\times\tfrac{1}{2}\right)\tilde{G}^A_{\mu\nu} G^{A\mu\nu} + \frac{g^2}{(4\pi)^2}\left(N_c \cdot \tfrac{1}{2}\right)\tilde{W}^a_{\mu\nu} W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2} 2N_c \left(\tfrac{1}{6}\right)^2 \tilde{B}_{\mu\nu} B^{\mu\nu}$$

$$\partial \cdot J_{\bar{u}_f} = \frac{g_s^2}{(4\pi)^2}\frac{1}{2}\tilde{G}^A_{\mu\nu} G^{A\mu\nu} + \frac{g'^2}{(4\pi)^2} N_c \left(\tfrac{2}{3}\right)^2 \tilde{B}_{\mu\nu} B^{\mu\nu}$$

$$\partial \cdot J_{\bar{d}_f} = \frac{g_s^2}{(4\pi)^2}\frac{1}{2}\tilde{G}^A_{\mu\nu} G^{A\mu\nu} + \frac{g'^2}{(4\pi)^2} N_c \left(\tfrac{-1}{3}\right)^2 \tilde{B}_{\mu\nu} B^{\mu\nu}$$

$$\partial \cdot J_\ell = + \frac{g^2}{(4\pi)^2}\frac{1}{2}\tilde{W}^a_{\mu\nu} W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2} 2\left(\tfrac{1}{2}\right)^2 \tilde{B}_{\mu\nu} B^{\mu\nu}$$

$$\partial \cdot J_{\bar{e}} = + \frac{g'^2}{(4\pi)^2} (+1)^2 \tilde{B}_{\mu\nu} B^{\mu\nu}$$

Sometimes convenient to express in terms of vector and axial currents.

$\longrightarrow$ break up currents into isospin components: insert $\frac{1}{2}(\underset{\uparrow}{1} \pm T^3)$ ← anomaly free (traceless)

only this contributes.

$$J^\mu_{q_f} = q^\dagger \bar\sigma^\mu q = u^\dagger \bar\sigma^\mu u + d^\dagger \bar\sigma^\mu d \equiv J^\mu_{u_f} + J^\mu_{d_f}$$

$$J^\mu_{l_f} = l^\dagger \bar\sigma^\mu l = \nu^\dagger \bar\sigma^\mu \nu + e^\dagger \bar\sigma^\mu e = J^\mu_{\nu_f} + J^\mu_{e_f}$$

Then anomalous contributions to divergence of <u>these</u> chiral currents are:

$$\partial\cdot J_{u_f} = \frac{g_s^2}{(4\pi)^2}\frac{1}{2}\tilde G^A_{\mu\nu}G^{A\mu\nu} + \frac{g^2}{(4\pi)^2}N_c\frac{1}{4}\tilde W^a_{\mu\nu}W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2}N_c\left(\frac{1}{6}\right)^2\tilde B_{\mu\nu}B^{\mu\nu}$$

$$\partial\cdot J_{d_f} = \frac{g_s^2}{(4\pi)^2}\frac{1}{2}\tilde G^A_{\mu\nu}G^{A\mu\nu} + \frac{g^2}{(4\pi)^2}N_c\frac{1}{4}\tilde W^a_{\mu\nu}W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2}N_c\left(\frac{1}{6}\right)^2\tilde B_{\mu\nu}B^{\mu\nu}$$

$$\partial\cdot J_{\nu_f} = + \frac{g^2}{(4\pi)^2}\frac{1}{4}\tilde W^a_{\mu\nu}W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2}\left(\frac{1}{2}\right)^2\tilde B_{\mu\nu}B^{\mu\nu}$$

$$\partial\cdot J_{e_f} = + \frac{g^2}{(4\pi)^2}\frac{1}{4}\tilde W^a_{\mu\nu}W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2}\left(\frac{1}{2}\right)^2\tilde B_{\mu\nu}B^{\mu\nu}$$

Now define vector and axial currents for <u>each fermion flavor</u>:

$$V: \begin{pmatrix} u \\ \bar u^\dagger \end{pmatrix} \longrightarrow e^{-i\alpha G}\begin{pmatrix} u \\ \bar u^\dagger \end{pmatrix} \qquad\qquad A: \begin{pmatrix} u \\ \bar u^\dagger \end{pmatrix} \longrightarrow e^{-i\alpha G\gamma_5}\begin{pmatrix} u \\ \bar u^\dagger \end{pmatrix}$$

$$J_V = \underbrace{J_\chi}_{\text{Left}} \underbrace{- J_{\bar\chi}}_{+\text{Right}} \qquad\qquad J_A = \underbrace{-J_\chi}_{-\text{Left}} \underbrace{- J_{\bar\chi}}_{+\text{Right}}$$

FOUR COMPONENT (vector) — FOUR COMPONENT (axial)

$$(J_V)^\mu_{u_f} = (u^\dagger\bar\sigma^\mu u - \bar u^\dagger \bar\sigma^\mu \bar u)_f = (\bar u\gamma^\mu u)_f \qquad (J_A)^\mu_{u_f} = -(u^\dagger\bar\sigma^\mu u + \bar u^\dagger\bar\sigma^\mu\bar u) = \bar u\gamma^\mu\gamma_5 u$$

$$(J_V)^\mu_{d_f} = (d^\dagger\bar\sigma^\mu d - \bar d^\dagger\bar\sigma^\mu\bar d)_f = (\bar d\gamma^\mu d)_f \qquad (J_A)^\mu_{d_f} = -(\quad) = \bar d\gamma^\mu\gamma_5 d$$

$$(J_V)^\mu_{\nu_f} = (\nu^\dagger\bar\sigma^\mu\nu - \underbrace{\bar\nu^\dagger\bar\sigma^\mu\bar\nu}_{\text{"dummy"}})_f = (\bar\nu\gamma^\mu\nu)_f \qquad (J_A)^\mu_{\nu_f} = -(\quad) = \bar\nu\gamma^\mu\gamma_5\nu$$

$$(J_V)^\mu_{e_f} = (e^\dagger\bar\sigma^\mu e - \bar e^\dagger\bar\sigma^\mu\bar e)_f = (\bar e\gamma^\mu e)_f \qquad (J_A)^\mu_{e_f} = -(\quad) = \bar e\gamma^\mu\gamma_5 e$$

Then the anomalous contributions to divergence of vector and axial-vector currents of a given generation f are:

VECTOR CURRENTS: (L+R)

$$(\partial\cdot J_V)_{u_f} = +\frac{g^2}{(4\pi)^2} N_c \frac{1}{4} \widetilde{W}^a_{\mu\nu} W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2} N_c \overbrace{\left[\left(\frac{1}{6}\right)^2 - \left(\frac{2}{3}\right)^2\right]}^{-5/12} \widetilde{B}_{\mu\nu} B^{\mu\nu}$$

$$(\partial\cdot J_V)_{d_f} = +\frac{g^2}{(4\pi)^2} N_c \frac{1}{4} \widetilde{W}^a_{\mu\nu} W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2} N_c \overbrace{\left[\left(\frac{1}{6}\right)^2 - \left(\frac{-1}{3}\right)^2\right]}^{-1/12} \widetilde{B}_{\mu\nu} B^{\mu\nu}$$

$$(\partial\cdot J_V)_{\nu_f} = +\frac{g^2}{(4\pi)} \frac{1}{4} \widetilde{W}^a_{\mu\nu} W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2} \left(\frac{1}{2}\right)^2 \widetilde{B}_{\mu\nu} B^{\mu\nu}$$

$$(\partial\cdot J_V)_{e_f} = +\frac{g^2}{(4\pi)^2} \frac{1}{4} \widetilde{W}^a_{\mu\nu} W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2} \underbrace{\left[\left(\frac{1}{2}\right)^2 - 1^2\right]}_{-3/4} \widetilde{B}_{\mu\nu} B^{\mu\nu}$$

AXIAL-VECTOR CURRENTS: (−L+R)

$$(\partial\cdot J_A)_{u_f} = \frac{-g_s^2}{(4\pi)^2} \widetilde{G}^A_{\mu\nu} G^{A\mu\nu} - \frac{g^2}{(4\pi)^2} N_c \frac{1}{4} \widetilde{W}^a_{\mu\nu} W^{a\mu\nu} - \frac{g'^2}{(4\pi)^2} N_c \overbrace{\left[\left(\frac{1}{6}\right)^2 + \left(\frac{2}{3}\right)^2\right]}^{17/36} \widetilde{B}_{\mu\nu} B^{\mu\nu}$$

$$(\partial\cdot J_A)_{d_f} = \frac{-g_s^2}{(4\pi)^2} \widetilde{G}^A_{\mu\nu} G^{A\mu\nu} - \frac{g^2}{(4\pi)^2} N_c \frac{1}{4} \widetilde{W}^a_{\mu\nu} W^{a\mu\nu} - \frac{g'^2}{(4\pi)^2} N_c \overbrace{\left[\left(\frac{1}{6}\right)^2 + \left(\frac{1}{3}\right)^2\right]}^{5/36} \widetilde{B}_{\mu\nu} B^{\mu\nu}$$

$$(\partial\cdot J_A)_{\nu_f} = -\frac{g^2}{(4\pi)^2} N_c \frac{1}{4} \widetilde{W}^a_{\mu\nu} W^{a\mu\nu} - \frac{g'^2}{(4\pi)^2} \left(\frac{1}{2}\right)^2 \widetilde{B}_{\mu\nu} B^{\mu\nu}$$

$$(\partial\cdot J_A)_{e_f} = -\frac{g^2}{(4\pi)^2} \frac{1}{4} \widetilde{W}^a_{\mu\nu} W^{a\mu\nu} - \frac{g'^2}{(4\pi)^2} \underbrace{\left[\left(\frac{1}{2}\right)^2 + 1^2\right]}_{5/4} \widetilde{B}_{\mu\nu} B^{\mu\nu}$$