Application: vector baryon and lepton currents

$\boxed{B}$ $\quad \psi_f \longrightarrow e^{-i\alpha/3}\,\psi_f \qquad\qquad \psi_f = \{u, d, s, c, b, t\}.$

$$J_B^\mu = \frac{1}{3}\left(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d + \bar{s}\gamma^\mu s + \bar{c}\gamma^\mu c + \bar{b}\gamma^\mu b + \bar{t}\gamma^\mu t\right)$$

$$\partial\cdot J_B = \frac{1}{3}\frac{g^2}{(4\pi)^2}\left[\overset{3}{\overset{\shortparallel}{n_f}}\times \overset{3}{\overset{\shortparallel}{N_c}}\times\frac{1}{4} + n_f N_c \times \frac{1}{4}\right]\tilde{W}^a_{\mu\nu}W^{a\mu\nu}$$

$$+\frac{1}{3}\frac{g'^2}{(4\pi)^2}\left[n_f N_c\left(\frac{-5}{12}\right) + n_f N_c\left(\frac{-1}{12}\right)\right]\tilde{B}_{\mu\nu}B^{\mu\nu}$$

$$= \frac{g^2}{(4\pi)^2}\,\frac{3}{2}\,\tilde{W}^a_{\mu\nu}W^{a\mu\nu} - \frac{g'^2}{(4\pi)^2}\,\frac{3}{2}\,\tilde{B}_{\mu\nu}B^{\mu\nu}$$

$\boxed{L}$ $\quad \psi_f \longrightarrow e^{-i\alpha}\,\psi_f \qquad\qquad \psi_f = \{\nu_e, e, \nu_\mu, \mu, \nu_\tau, \tau\}.$

$$J_L^\mu = \left(\bar{\nu}_e\gamma^\mu\nu_e + \bar{e}\gamma^\mu e + \bar{\nu}_e\gamma^\mu\nu_e + \bar{\mu}\gamma^\mu\mu + \bar{\nu}_\tau\gamma^\mu\nu_\tau + \bar{\tau}\gamma^\mu\tau\right)$$

$$\partial\cdot J_L = \frac{g^2}{(4\pi)^2}\left[n_f\frac{1}{4} + n_f\frac{1}{4}\right]\tilde{W}^a_{\mu\nu}W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2}\left[n_f\frac{1}{4} - n_f\frac{3}{4}\right]\tilde{B}_{\mu\nu}B^{\mu\nu}$$

$$= \frac{g^2}{(4\pi)^2}\,\frac{3}{2}\,\tilde{W}^a_{\mu\nu}W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2}\,\frac{3}{2}\,\tilde{B}_{\mu\nu}B^{\mu\nu}$$

$\boxed{B+L}$ $\quad J^\mu_{B+L} = J^\mu_B + J^\mu_L$

$$\partial\cdot J_{B+L} = \frac{g^2}{(4\pi)^2}\,3\,\tilde{W}^a_{\mu\nu}W^{a\mu\nu} + \frac{g'^2}{(4\pi)^2}\,3\,\tilde{B}_{\mu\nu}B^{\mu\nu}$$

$\boxed{B-L}$ $\quad J^\mu_{B-L} = J^\mu_B - J^\mu_L$

$\partial\cdot J^\mu_{B-L} = 0 \qquad$ [Anomaly free]

Integration over $\underline{\partial_\mu J^\mu_{B+L}}$:

$$\int_{-\infty}^{\infty} dt \int d^3\vec{x} \;\; \partial_\mu J^\mu_{B+L} = \int d^4x \;\; n_f \left(\frac{g^2}{8\pi^2} \, \mathrm{Tr}[W_{\mu\nu}\tilde{W}^{\mu\nu}] - \frac{g'^2}{16\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu} \right)$$

$$\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}[F_{\mu\nu}F_{\rho\sigma}] = \mathrm{Tr}[F_{\mu\nu}\tilde{F}^{\mu\nu}] = \frac{1}{2}\partial_\mu K^\mu$$

where $K^\mu = 4\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}[A_\nu \partial_\rho A_\sigma + \frac{2}{3} i g A_\nu A_\rho A_\sigma]$

Then: $\displaystyle \partial_\mu J^\mu_{B+L} = n_f \left(\frac{g^2}{16\pi^2}\partial_\mu K^\mu - \frac{g'^2}{16\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu} \right)$

$$\int_{-\infty}^{\infty} dt \int d^3\vec{x} \;\; \partial_\mu J^\mu_{B+L} = \int d^4x \;\; n_f \left(\frac{g^2}{16\pi^2}\partial_\mu K^\mu - \frac{g'^2}{16\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu} \right)$$

$$= 2 n_f \, \nu_{cs}$$

one lepton
and one baryon (three quarks)