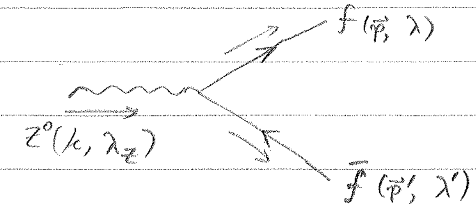
Z^0 width

$$\boxed{Z^0 \longrightarrow f\bar{f}}$$



Kinematics:

$k^\mu = (p+p')^\mu$

$k^2 = m_Z^2$

$p^2 = p'^2 = 0$

$(k-p')^2 = p^2$

$M_Z^2 - 2k\cdot p' + 0 = 0$

$\Rightarrow k\cdot p' = \frac{1}{2}M_Z^2 = k\cdot p$

also

$M_Z^2 = \cancel{p^2 + p'^2} + 2p\cdot p'$

Approximate: massless fermions

— can compute partial widths for $f_L \bar{f}_R$ and $f_R \bar{f}_L$ separately.

$Z^0 \to f_L \bar{f}_R$
$$i\mathcal{M}_L = \bar{u}(\vec{p}, \lambda)\left[-ig_Z \gamma^\mu \underbrace{(T_f^3 - Q_f \sin^2\theta_W)}_{g_L^f} \hat{P}_L\right] v(\vec{p}', \lambda')\, \epsilon_\mu(\vec{k}, \lambda_Z)$$

$Z^0 \to f_R \bar{f}_L$
$$i\mathcal{M}_R = \bar{u}(\vec{p}, \lambda)\left[-ig_Z \gamma^\mu \underbrace{(-Q_f \sin^2\theta_W)}_{g_R^f} \hat{P}_R\right] v(\vec{p}', \lambda')\, \epsilon_\mu(\vec{k}, \lambda_Z)$$

Both cases: Denote $\lambda = \{L, R\}$ as helicity of outgoing fermion f.

$$i\mathcal{M}_\lambda = \bar{u}(\vec{p}, \lambda)\left[-ig_Z \gamma^\mu g_\lambda^f \hat{P}_\lambda\right] v(\vec{p}', s)\, \epsilon_\mu(\vec{k}, \lambda_Z).$$

square amplitude:

average over inital Z^0 helicity states. $\frac{1}{3}\sum_{\lambda_Z}$

safe to sum over final fermion helicites $\sum_{ss'}$

sum over quark colors. $\sum_{color} = 3$ for quarks.

$$|\overline{\mathcal{M}}|_\lambda^2 = g_Z^2 g_\lambda^2\, N_C \left(\sum_{s,s'}{}' \bar{u}_{p's}\, \gamma^\mu \hat{P}_\lambda\, v_{p's'}\, \bar{v}_{p's'}\, \gamma^\nu \hat{P}_\lambda\, u_{p,s}\right)\left(\frac{1}{3}\sum_{\lambda_Z} \epsilon_\mu \epsilon_\nu^*\right)$$

$$= g_Z^2 g_\lambda^2 \frac{N_C}{3} \mathrm{Tr}\left[\not{p}\, \gamma^\mu \hat{P}_\lambda\, \not{p}' \gamma^\nu \hat{P}_\lambda\right]\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_Z^2}\right)$$

$$\mathrm{Tr}\left[\not{p}\, \gamma^\mu \not{p}' \gamma^\nu \underbrace{\hat{P}_\lambda \hat{P}_\lambda}_{P_\lambda}\right]\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_Z^2}\right)$$

part antisymmetric in $\mu \longleftrightarrow \nu$ vanishes because $(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_Z^2})$ is symmetric. $\Rightarrow$ Trace indep. of λ.

$$|\overline{\mathcal{M}}|^2_\lambda = g_Z^2 \, g_\lambda^2 \, \frac{N_c}{3} \cdot \left(2 m_Z^2\right)$$ indep. of angles.

$\longrightarrow$ Partial with:

$$d\Gamma_{f_\lambda} = \frac{1}{2m_Z} |\overline{\mathcal{M}}|^2_\lambda \cdot \frac{1}{32\pi^2} d\Omega$$

can be immediately integrated:

$$\Gamma_{f_\lambda} = \frac{1}{2m_Z} |\overline{\mathcal{M}}|^2_\lambda \cdot \frac{1}{8\pi}$$

$$= \frac{1}{16\pi m_Z} \, g_Z^2 \, g_\lambda^2 \, \frac{N_c}{3} \cdot 2 m_Z^2$$

$$\boxed{\Gamma_{f_\lambda} = \frac{g_Z^2}{24\pi} (g_\lambda^f)^2 N_c \cdot m_Z}$$

Partial width for $Z^0 \to f_L \bar{f}_R$.

	channel:	Γ_λ		incl. 2nd & 3rd gen.	includes
$\boxed{f_L \bar{f}_R}$	$\bar{u}_L u_R$	$\frac{g_Z^2 m_Z}{24\pi} \cdot 3\left(\frac{1}{2} - \frac{2}{3} s_\theta^2\right)^2$	≈ 0.24 GeV	$\xrightarrow{\times 2}$ 0.48 GeV	$\leftarrow$ $u\bar{u}$, $c\bar{c}$
	$\bar{d}_L d_R$	$\frac{g_Z^2 m_Z}{24\pi} \cdot 3\left(\frac{-1}{2} + \frac{1}{3} s_\theta^2\right)^2$	≈ 0.35	$\xrightarrow{\times 3}$ 1.07 GeV	$\leftarrow$ $d\bar{d}$, $s\bar{s}$, $b\bar{b}$
	$\bar{\nu}_L \nu_R$	$\frac{g_Z^2 m_Z}{24\pi} \cdot \left(\frac{1}{2}\right)^2$	≈ 0.17	$\longrightarrow$ 0.50 GeV	$\leftarrow$ $\nu_e\bar{\nu}_e$, $\nu_\mu\bar{\nu}_\mu$, $\nu_\tau\bar{\nu}_\tau$
	$\bar{e}_L e_R$	$\frac{g_Z^2 m_Z}{24\pi} \cdot \left(\frac{-1}{2} + s_\theta^2\right)^2$	≈ 0.048	$\longrightarrow$ 0.14 GeV	$\leftarrow$ e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$
$\boxed{f_R \bar{f}_L}$	$u_R \bar{u}_L$	$\frac{g_Z^2 m_Z}{24\pi} \cdot 3\left(-\frac{2}{3} s_\theta^2\right)^2$	≈ 0.047	$\xrightarrow{\times 2}$ 0.094 GeV	
	$d_R \bar{d}_L$	$\frac{g_Z^2 m_Z}{24\pi} \cdot 3\left(\frac{1}{2} s_\theta^2\right)^2$	≈ 0.012	$\xrightarrow{\times 3}$ 0.035 GeV	
	$\nu_R \bar{\nu}_L$	$\frac{g_Z^2 m_Z}{24\pi} \cdot 0$	$= 0$	0	
	$e_R \bar{e}_L$	$\frac{g_Z^2 m_Z}{24\pi} \cdot \left(s_\theta^2\right)^2$	≈ 0.035	$\longrightarrow$ 0.106 GeV	

modes related by parity

$$\boxed{\Gamma_Z = 2.42 \text{ GeV}}$$