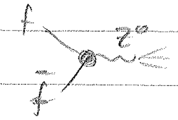
Left-right cross section asymmetry: — Measured at SLC

Let σ_L^f be Z-boson production cross section with left-handed initial fermion
Let σ_R^f be Z-boson " " " " right-handed initial fermion



L-R asymmetry defined by:

$$A_{LR}^{(f)} = \frac{\sigma_L^f - \sigma_R^f}{\sigma_L^f - \sigma_R^f}$$ — characterizes $Z^0 f\bar{f}$ chiral coupling.

$\boxed{\text{LONG AND CLUMSY DERIVATION}}$

$$\sigma_L^f = \sum_{\lambda_{final}}' \sigma_{\lambda_f \leftarrow \lambda_i = -1/2} = \frac{N_c}{12\pi} s \left(|G_{\frac{1}{2} \leftarrow -\frac{1}{2}}(s)|^2 + |G_{\frac{1}{2} \leftarrow -\frac{1}{2}}(s)|^2 \right)_{s = M_Z^2}.$$

$$= \frac{N_c}{12\pi} M_Z \left(|G_{L\leftarrow L}(M_Z)|^2 + |G_{R\leftarrow L}(M_Z)|^2 \right)$$

similarly,

$$\sigma_R^f = \sum_{\lambda_{final}}' \sigma_{\lambda_f \leftarrow \lambda_i = +1/2} = \frac{N_c}{12\pi} M_Z \left(|G_{L\leftarrow R}(M_Z)|^2 + |G_{R\leftarrow R}(M_Z)|^2 \right)$$

Then:

$$A_{LR}^{(f)} = \frac{|G_{L\leftarrow L}(M_Z)|^2 + |G_{R\leftarrow L}(M_Z)|^2 - |G_{L\leftarrow R}(M_Z)|^2 - |G_{R\leftarrow R}(M_Z)|^2}{|G_{L\leftarrow L}(M_Z)|^2 + \quad '' \quad + \quad '' \quad + \quad ''}.$$

QED contribution (γ exchange) negligible. $|G|^2 \sim \underbrace{\frac{g^4}{\cos^4\theta_W} \frac{1}{M_Z^2 \Gamma^2}}_{\text{divides away.}} (g_\lambda^{f'})^2 (g_\lambda^f)^2$

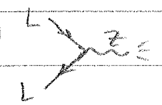
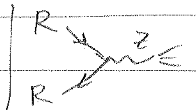
$$A_{LR}^{(f)} = \frac{(T_{f'}^3 - Q_{f'} s_W^2)^2 (T_f^3 - Q_f s_W^2) + (-Q_{f'} s_W^2)^2 (T_f^3 - Q_f s_W^2) - (T_{f'}^3 - Q_{f'} s_W^2)^2 (-Q_f s_W^2)^2 - (-Q_{f'} s_W^2)(\ )}{\quad + \quad\quad + \quad}$$

$$= \frac{\left[(T_{f'}^3 - Q_{f'} s_W^2)^2 + (-Q_{f'} s_W^2)^2\right] (T_i^3 - Q_i s_W^2)^2 - \left[(T_{f'}^3 - Q_{f'} s_W^2)^2 + (-Q_{f'} s_W^2)^2\right] (-Q_i s_W^2)^2}{\quad '' \quad + \quad '' \quad}$$

$$= \frac{\cancel{\left[(T_{f'}^3 - Q_{f'} s_W^2)^2 + (-Q_{f'} s_W^2)^2\right]} \left[(T_i^3 - Q_i s_W^2)^2 - (-Q_i s_W^2)^2\right]}{\cancel{\left[\quad\quad\right]} \left[\quad + \quad\right]}$$

$\boxed{\text{QUICK AND CLEANER DERIVATION}}$

Essentially, the L/R asymmetry characterizes the difference between:

$\Big|$  $\Big|^2$ and $\Big|$  $\Big|^2$

– main point is that the <u>only</u> difference in these two production cross sections is the <u>coupling</u>. All kinematic/spin dependence is common.
(because electron mass can be neglected)

$$\mathcal{M}_{L/R\to Z} = \bar{v}_{p'\lambda'}\left[-ig_Z\gamma^\mu g^e_{L/R}\,\hat{P}_{L/R}\right]u_{p,\lambda}\,\epsilon^*_\mu(\vec{k},\lambda_Z)$$

$$= -ig_Z\, g^e_{L/R}\left(\bar{v}_{p'\lambda'}\gamma^\mu\hat{P}_{L/R}\,u_{p,\lambda}\right)\epsilon^*_\mu(\vec{k},\lambda_Z)$$

Square: sum over initial and final helicities

$$\left|\overline{\mathcal{M}}_{L/R\to Z}\right|^2 = g_Z^2\,(g^e_{L/R})^2\,\underbrace{\mathrm{Tr}\left[\not{p}'\gamma^\mu\hat{P}_{L/R}\gamma^\mu\hat{P}_{L/R}\right]\left(-g_{\mu\mu'}+\frac{k_\mu k_{\mu'}}{m_Z}\right)}$$

$p^2 = p'^2 = 0,$

$p\cdot p' = \frac{1}{2}s \simeq \frac{1}{2}m_Z^2$ ← on Z pole

$$= g_Z^2\,(g^e_{L/R})^2\cdot 2m_Z^2 \qquad \text{[Indep. of chirality]}$$

Then cross section:

$$\sigma_{L/R} \sim \frac{1}{2s}\left|\overline{\mathcal{M}}_{L/R\to Z}\right|^2 d(\mathrm{LIPS})_1$$

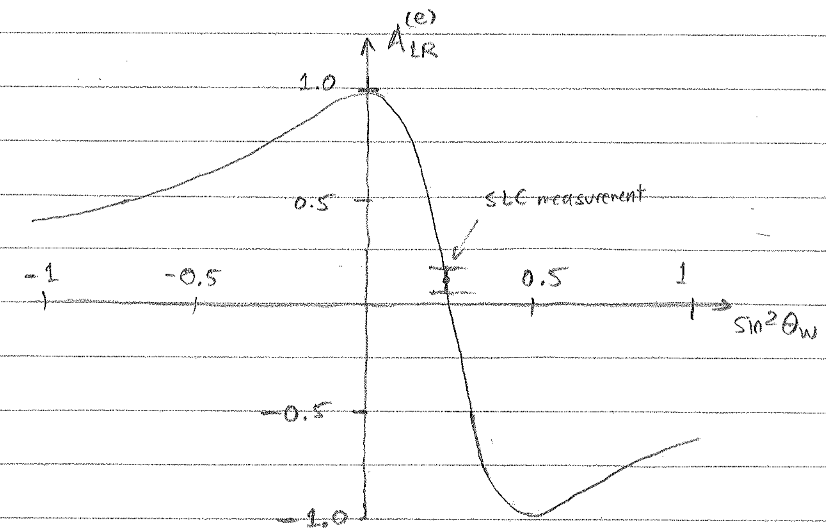
$$\therefore\ \boxed{\begin{aligned}\sigma_L &\propto g_L^2 = \left(-\tfrac{1}{2}+\sin^2\theta_W\right)^2\\ \sigma_R &\propto g_R^2 = \left(0+\sin^2\theta_W\right)^2\end{aligned}}$$

So

$$A_{LR}^{(e)} = \frac{(-\frac{1}{2}+s_W^2)^2 - (s_W^2)^2}{(-\frac{1}{2}+s_W^2)^2 + (s_W^2)^2} = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}$$

$$= \frac{1/4 + \cancel{s_W^4} - s_W^2 - \cancel{s_W^4}}{1/4 + s_W^4 - s_W^2 + s_W^4}$$

$$= \frac{1 - 4s_W^2}{1 - 4s_W^2 + 4s_W^4} = \frac{2(1-4s_W^2)}{1+(1-4s_W^2)} = \frac{2g_V g_A}{g_V^2 + g_A^2}$$



$\theta_W = 0$ leads to largest asymmetry (couples only to e_L)
$\sin^2\theta_W = 1/4$ leads to zero asymmetry — highest sensitivity to $\sin^2\theta_W$.

- The small value of A_{LR} can be ascribed to the nearly vanishing <u>vector</u> coupling to the Z-boson $g_V^{(e)} = -0.0375$.

In the limit $g_V^{(e)} \to 0$, the coupling is purely axial — if Z is "assigned" $\underline{1^+}$ (axial-vector), then e^+e^-Z interactions become <u>parity conserving</u>.

Unpolarized differential cross section:

Average over incoming polarization $\frac{1}{4}\sum_{\lambda_e}$

Sum over final polarizations $\sum_{\lambda_f}$

$$\therefore \quad \frac{d\bar{\sigma}}{d\cos\theta} = \frac{s}{32\pi} N_c \, \frac{1}{4} \sum_{\lambda_e \lambda_f} \left| G_{\lambda_f \leftarrow \lambda_e}(s) \right|^2 \left[1 + \cos^2\theta + 8\lambda_e \lambda_f \cos\theta \right]$$

$$= \frac{s}{128\pi} N_c \Bigg[\left(\left|G_{\frac{1}{2}\leftarrow\frac{1}{2}}(s)\right|^2 + \left|G_{\frac{1}{2}\leftarrow-\frac{1}{2}}(s)\right|^2 + \left|G_{-\frac{1}{2}\leftarrow\frac{1}{2}}(s)\right|^2 + \left|G_{-\frac{1}{2}\leftarrow-\frac{1}{2}}(s)\right|^2 \right)(1+\cos^2\theta)$$

$$+ 2\left(\left|G_{\frac{1}{2}\leftarrow\frac{1}{2}}(s)\right|^2 - \left|G_{\frac{1}{2}\leftarrow-\frac{1}{2}}(s)\right|^2 - \left|G_{-\frac{1}{2}\leftarrow\frac{1}{2}}(s)\right|^2 + \left|G_{-\frac{1}{2}\leftarrow-\frac{1}{2}}(s)\right|^2 \right) \cos\theta \Bigg]$$

Unpolarized cross section:

$$\bar{\sigma} = \int_{-1}^{1} d\cos\theta \, \frac{d\bar{\sigma}}{d\cos\theta}$$ term odd in $\cos\theta$ integrates to 0.

$$= \frac{s}{128\pi^2} N_c \left(|G_{R\leftarrow R}(s)|^2 + |G_{R\leftarrow L}(s)|^2 + |G_{L\leftarrow R}(s)|^2 + |G_{L\leftarrow L}(s)|^2 \right) \times \frac{8}{3}.$$

$$= \frac{s}{24\pi^2} N_c \left(\quad '' \quad\quad '' \quad\quad '' \quad\quad '' \quad \right)$$