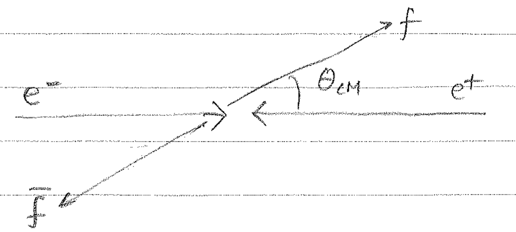
Forward-backward asymmetry: Measured at LEP



initial state unpolarized
final state polarization unmeasured.

Let $N_F \equiv$ number of f scattered in forward direction $0 \leq \theta \leq \frac{\pi}{2}$

Let $N_B \equiv$ " " f " " backward " $\frac{\pi}{2} < \theta \leq \pi$.

Then forward-backward asymmetry defined by:

$$A_{FB}^{f} = \frac{N_F - N_B}{N_F + N_B} \sim \frac{\sigma_F - \sigma_B}{\sigma_F - \sigma_B} \qquad \text{[UNPOLARIZED]}$$

Defined for each particle species f that is not initial state (e).

Theoretical expectation:

$$\frac{d\bar{\sigma}}{d\cos\theta} = \sum_{\lambda_e, \lambda_f} \frac{m_Z}{32\pi} N_c^f \left(\frac{g^4}{c_W^4} \frac{(g_\lambda^f g_\lambda^e)^2}{m_Z^2 \Gamma_Z} \right) \left[1 + \cos^2\theta + 8 \lambda_e \lambda_f \cos\theta \right]$$

$$= \frac{1}{32\pi} N_c^f \frac{g_Z^4}{m_Z \Gamma_Z^2} \Big[\underbrace{(g_L^e + g_R^e)(g_L^f + g_R^f)}_{\text{factor out}} (1 + \cos^2\theta) + (g_L^e - g_R^e)(g_L^f - g_R^f) \times 2\cos\theta \Big]$$

$$= \frac{1}{32\pi} N_c^f \frac{g_Z^4}{m_Z \Gamma_Z^2} (g_L^e + g_R^e)(g_L^f + g_R^f) \left[1 + \cos^2\theta + \frac{(g_L^e - g_R^e)(g_L^f - g_R^f)}{(g_L^e + g_R^e)(g_L^f + g_R^f)} \times 2\cos\theta \right] \qquad (*)$$

Then integrate over scattering angles:

Forward:

$$\int_0^1 d\cos\theta \left[1 + \cos^2\theta \right] = \frac{4}{3}$$

$$\int_0^1 d\cos\theta \left[\cos\theta \right] = \frac{1}{2}$$

Backward:

$$\int_{-1}^0 d\cos\theta \left[1 + \cos^2\theta \right] = \frac{4}{3}$$

$$\int_{-1}^0 d\cos\theta \left[\cos\theta \right] = -\frac{1}{2}$$

$$\sigma_F = \int_0^1 d\cos\theta \, \frac{d\sigma}{d\cos\theta} \propto \left[\frac{4}{3} + \frac{(g_L^e - g_R^e)(g_L^f - g_R^f)}{(g_L^e + g_R^e)(g_L^f + g_R^f)} \, 2 \times \frac{1}{2} \right]$$

$$\sigma_B = \int_0^1 d\cos\theta \, \frac{d\sigma}{d\cos\theta} \propto \left[\frac{4}{3} - \frac{(g_L^e - g_R^e)(g_L^f - g_R^f)}{(g_L^e + g_R^e)(g_L^f - g_R^f)} \, 2 \times \frac{1}{2} \right]$$

$$\therefore \quad A_{FB}^f = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{2 \times \frac{(g_L^e - g_R^e)(g_L^f - g_R^f)}{(g_L^e + g_R^e)(g_L^f + g_R^f)}}{8/3}$$

$$= \frac{3}{4} \, \frac{g_L^e - g_R^e}{g_L^e + g_R^e} \, \frac{g_L^f - g_R^f}{g_L^f + g_R^f} = \frac{3}{4} A_{LR}^{(e)} A_{LR}^{(f)}$$

Experimental measurement:

- Typically detectors do not quite have complete coverage of polar scattering angles.
- It is better to fit curve in $\cos\theta$ to data.

Notice: $\frac{d\sigma}{d\cos\theta}$ in (*) can be written,

$$\frac{d\sigma}{d(\cos\theta)} = \underbrace{\frac{1}{32\pi} N_c^f \frac{g_Z^4}{m_Z \Gamma_Z^2} (g_L^e + g_R^e)(g_L^f + g_R^f)}_{\mathcal{N}} \left[1 + \cos^2\theta + \frac{8}{3} A_{FB}^f \cos\theta \right]$$

Fit to two parameters: $\mathcal{N}$ & A_{FB}.