Higgs Sector

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mathcal{L}_{Higgs} = (D_\mu H)^\dagger (D^\mu H) - V(H)$$

$Y = 1/2$

where $D_\mu H = (\partial_\mu + igT^a W_\mu^a + ig' Y B_\mu) H$ and

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$\mathcal{L}_{Higgs} = \left[H^\dagger (\overleftarrow{\partial}_\mu - igT^a W_\mu^a - ig' Y B_\mu)(\overrightarrow{\partial}^\mu + igT^a W^{\mu a} + ig' Y B^\mu) H \right] - \left[-\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \right]$$

POTENTIAL TERMS:

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$= \frac{-\mu^2}{2} (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) + \frac{\lambda}{4} (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)^2$$

This potential is totally symmetric with respect to a (global) SO(4) symmetry.
$\Longrightarrow$ could parametrize as one radial and three angular fields "unitary parametrization".
– dangerous; see later.

By convention ϕ_3 acquires an expectation value. $\longleftarrow$ doesn't matter which component acquires VEV.
(though any linear combination would work)

$\boxed{\text{Minimization condition}}$: $\left. \frac{\partial V}{\partial \phi_3} \right|_{\phi_3 = v} = -\mu^2 \phi_3 + \lambda \phi_3^3 \Big|_{\phi_3 = v} = 0$

$$\Rightarrow v = \left\{ \pm \sqrt{\mu^2/\lambda},\ 0 \right\}$$

$\boxed{\text{Stability condition}}$: $\left. \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \right|_{\substack{\phi_1 = \phi_2 = \phi_4 = 0 \\ \phi_3 = \{\pm\sqrt{\mu^2/\lambda},\, 0\}}} = m_{ij}^2 (v = 0) = \begin{pmatrix} -\mu^2 & & & \\ & -\mu^2 & & \\ & & -\mu^2 & \\ & & & -\mu^2 \end{pmatrix}$ unstable

or

$$= m_{ij}^2 (v = \pm\sqrt{\mu^2/\lambda}) = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 2\lambda v^2 & \\ & & & 0 \end{pmatrix}$$ stable

$m_H^2 \equiv$ Higgs mass.

Quantize system in background of Higgs vacuum expectation value (VEV) $\quad v = +\sqrt{\mu^2/\lambda}$

- done purely for sake of perturbation theory.

$$H(x) = \langle H \rangle + H'(x) \qquad\qquad \frac{1}{\sqrt{2}}(\phi_1 \pm i\phi_2) = \phi^{\pm}(x)$$

$$= \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{pmatrix} + \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(h + i\phi^0) \end{pmatrix} \qquad\qquad \phi_3 = \underset{\text{VEV}}{v} + h(x)$$

$$= \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + h + i\phi^0) \end{pmatrix} \qquad\qquad \phi_4 = \phi^0(x)$$

Definitions and relations

inversion relations:

$$v = +\sqrt{\mu^2/\lambda} \qquad\qquad \lambda = m_H^2/2v^2$$

$$m_H^2 = 2\lambda v^2 = 2\mu^2 \qquad\qquad \mu = m_H/\sqrt{2}$$

POTENTIAL TERMS

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

eliminate μ^2 in favor of v^2 and λ.

$$V(H) = -\frac{1}{4}\lambda v^4 + \frac{1}{2}\overbrace{(2\lambda v^2)}^{m_H^2} h^2 + \lambda v h^3 + \lambda v\, h\phi^0\phi^0 + 2\lambda h \phi^+\phi^-$$

$$+ \frac{\lambda}{4} h^4 + \frac{1}{2}\lambda h^2 \phi^0\phi^0 + \lambda h^2 \phi^+\phi^-$$

$$+ \frac{1}{4}\lambda (\phi^0)^4 + \lambda \phi^0\phi^0\phi^+\phi^- + \lambda(\phi^+\phi^-)^2$$