Reduction to Fermi's theory of weak interactions

SM generating functional (no gauge fixing*)

$$Z = \int \mathcal{D}W^{+} \mathcal{D}Z \ldots \; e^{i\int d^4x\, \mathcal{L}_{SM}}$$

with
$$\mathcal{L}_{SM} = -\frac{1}{2} W^{+}_{\mu\nu} W^{\mu\nu -} + m_W^2 W^{+}\cdot W^{-} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + m_Z^2 Z\cdot Z$$
$$-\frac{g}{2\sqrt{2}}\left(W^{+}\cdot J^{-} - W^{-} J^{+}\right) - \frac{g_Z}{2} Z\cdot J_{NC} + \ldots$$

Rewrite kinetic term for gauge bosons:

$$-\frac{1}{2} W^{+}_{\mu\nu} W^{\mu\nu -} + m_W^2 W^{+}\cdot W^{-} = W^{+}_{\mu} \underbrace{\left((\partial^2 - m_W^2) g^{\mu\nu} - \partial^\mu \partial^\nu\right)}_{O_W} W^{-}_{\nu} + \text{surface}$$

$$-\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{2} m_Z^2 Z\cdot Z = \frac{1}{2} Z_\mu \underbrace{\left((\partial^2 - m_Z^2) g^{\mu\nu} - \partial^\mu \partial^\nu\right)}_{O_Z} Z_\nu + \text{surface.}$$

Perform Gaussian integral over $W^{\pm}$ & Z

$$Z = \int \mathcal{D}\Phi_{SM} \underbrace{\frac{1}{\det O_W} \frac{1}{\sqrt{\det O_Z}}}_{\text{overall normalization}} e^{i\int d^4x\, d^4y \left[-\left(\frac{g}{2\sqrt{2}}\right)^2 J^{+}_x (O_W^{-1})_{xy} J^{-}_y - \frac{1}{2}\left(\frac{g_Z}{2}\right)^2 J^{NC}_x (O_W^{-1})_{xy} J^{NC}_y + \ldots\right.}$$

$$O^{-1}_{W \text{ or } Z} = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-y)} \frac{-1}{p^2 - m^2}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2}\right)$$

low energy approximation

$$\approx \int \frac{d^4p}{(2\pi)^d} e^{ip\cdot(x-y)} \frac{g^{\mu\nu}}{m^2}$$

$$\approx \delta^{(4)}(x-y) \frac{g^{\mu\nu}}{m^2}$$

Integrate over y, fixing $y \to x$

$$Z = \mathcal{N} \int \mathcal{D}\Phi_{SM}\; e^{i\int d^4x \left[-\frac{g^2}{8m^2} J^{+}\cdot J^{-} - \frac{g_Z^2}{8m_Z^2} J_{NC}\cdot J_{NC} + \ldots\right]}$$

Insert $m_W = \frac{1}{2} g v$, $m_Z = \frac{1}{2} g_Z v$

$$\mathcal{L}_{eff} = \frac{-1}{2v^2} J^+ \cdot J^- - \frac{1}{2v^2} J_{NC} \cdot J_{NC} + \ldots$$

Q: In $g \to 0$ limit, weak interactions survive ?!

A: In that limit $M_W, M_Z \to 0$ and low energy approx. not valid.

Match against Fermi's theory of weak interactions:

$$\mathcal{L}_{Fermi} = \frac{-G_F}{\sqrt{2}} J^+ \cdot J^-$$

$$\Rightarrow G_F = \frac{1}{\sqrt{2} v^2}$$

$$\therefore \quad \mathcal{L}_{eff} = \frac{-G_F}{\sqrt{2}} J^+ \cdot J^- - \frac{G_F}{\sqrt{2}} J_{NC} \cdot J_{NC} + \ldots$$

Effective theory valid provided $Q^2 \ll M_W^2$

Same strength! (of course, depends on normalization of currents)