Yukawa interactions

$$\mathcal{L}_{\text{Yuk}} = -\tilde{H}\cdot q_f^\dagger (y_u)_{fg}\, \bar{u}_g^\dagger - H\cdot q_f^\dagger (y_d)_{fg}\, \bar{d}_g^\dagger - H\cdot \ell_f^\dagger (y_\ell)_{fg}\, \bar{e}_g^\dagger + h.c.$$

$\tilde{H} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$

Upon diagonalizing the Yukawa matrices (at the expense of off-diagonal charged current interactions),

$$= -\begin{pmatrix} u^\dagger \\ d^\dagger \end{pmatrix}_f \cdot \left(\mathbb{1}_{fg}\,\phi^{0*},\ (V^\dagger_{CKM})_{fg}(-\phi^-) \right) \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix}_{gh} \cdot \bar{u}_h^\dagger + h.c.$$

$$- \begin{pmatrix} u^\dagger \\ d^\dagger \end{pmatrix}_f \cdot \left((V_{CKM})_{fg}\,\phi^+,\ \mathbb{1}_{fg}\,\phi^0 \right) \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix}_{gh} \cdot \bar{d}^\dagger + h.c.$$

$$- \begin{pmatrix} \nu^\dagger \\ e^\dagger \end{pmatrix}_f \cdot \left(\phi^+,\ \phi^0 \right) \begin{pmatrix} y_e & & \\ & y_\mu & \\ & & y_\tau \end{pmatrix}_{fg} \bar{e}_g^\dagger + h.c.$$

Open up dot product:

$$= \left[-(y_u)_{fg}\,\phi^{0*}\, u_f^\dagger \bar{u}_g^\dagger + h.c. \right] + \left[(V^\dagger_{CKM}\, y_u)_{fg}\,\phi^-\, d_f^\dagger \bar{u}_g^\dagger + h.c. \right]$$

$$+ \left[-(V_{CKM}\, y_d)_{fg}\,\phi^+\, u_f^\dagger \bar{d}_g^\dagger + h.c. \right] + \left[-(y_d)_{fg}\,\phi^0\, d_f^\dagger \bar{d}_g^\dagger + h.c. \right]$$

$$+ \left[-(y_\ell)_{fg}\,\phi^+\, \nu_f^\dagger \bar{e}_g^\dagger + h.c. \right] + \left[-(y_\ell)_{fg}\,\phi^0\, e_f^\dagger \bar{e}_g^\dagger + h.c. \right]$$

<u>Shift Higgs</u>:

$$\phi^0 \to \frac{1}{\sqrt{2}}(v + h + i\phi_z)$$
$$\phi^{0*} \to \frac{1}{\sqrt{2}}(v + h + i\phi_z)$$

explicitly write out h.c. terms:

use $(a_i V_{ij} b_j)^\dagger = b_j^* (V^\dagger)_{ji} a_i^* = b_i^* (V^\dagger)_{ij} a_j^*$

$(V_{ij}\, a_i b_j)^\dagger \qquad = (V^\dagger)_{ij}\, b_i^* a_j^*$

$$= \left[-(y_u)_{fg} \tfrac{1}{\sqrt{2}}(v + h - i\phi_Z)\, u_f^\dagger \bar{u}_g^\dagger - (y_u)_{fg} \tfrac{1}{\sqrt{2}}(v + h + i\phi_Z)\, u_f \bar{u}_g \right]$$

$$+ \left[(V_{CKM}^\dagger y_u)_{fg}\, \phi^- d_f^\dagger \bar{u}_g^\dagger + (y_u V_{CKM})_{fg}\, \phi^+ \bar{u}_f d_g \right]$$

combine.

$$+ \left[-(V_{CKM} y_d)_{fg}\, \phi^+ u_f^\dagger \bar{d}_g^\dagger - (y_d V_{CKM}^\dagger)_{fg}\, \phi^- \bar{d}_f u_g \right]$$

$$+ \left[-(y_d)_{fg} \tfrac{1}{\sqrt{2}}(v + h + i\phi_Z)\, d_f^\dagger \bar{d}_g^\dagger - (y_d)_{fg} \tfrac{1}{\sqrt{2}}(v + h - i\phi_Z)\, d_f \bar{d}_g \right]$$

$$+ \left[-(y_\ell)_{fg}\, \phi^+ \nu_f^\dagger \bar{e}_g^\dagger + (y_\ell)_{fg}\, \phi^- \bar{e}_f \nu_g \right] + \left[(y_\nu)_{fg}\, \phi^- e_f^\dagger \bar{\nu}_g^\dagger + (y_\nu)_{fg}\, \phi^+ \bar{\nu}_f e_g \right]$$

$$+ \left[-(y_\ell)_{fg} \tfrac{1}{\sqrt{2}}(v + h + i\phi_Z)\, e_f^\dagger \bar{e}_g^\dagger - (y_\ell)_{fg} \tfrac{1}{\sqrt{2}}(v + h - i\phi_Z)\, e_f \bar{e}_g \right]$$

$$d_f^\dagger (V^\dagger y_u)_{fg} \bar{u}_g^\dagger - \bar{d}_f (V^\dagger y_d)_{fg} u_g = \bar{d}\,(V^\dagger y_u P_R + V^\dagger y_d P_L)$$

Convert to 4-component spinors:

$$= \left[-(y_u)_{fg} \tfrac{1}{\sqrt{2}} v\, \bar{u}_f u_g - (y_u)_{fg} \tfrac{1}{\sqrt{2}} h\, \bar{u}_f u_g + i (y_u)_{fg} \tfrac{1}{\sqrt{2}} \phi_Z\, \bar{u}_f \gamma_5 u_g \right]$$

$$+ \left[\phi^- \bar{d}_f \left((V_{CKM}^\dagger y_u)_{fg} \hat{P}_R - (y_d V_{CKM}^\dagger)_{fg} \hat{P}_L \right) u_g + \phi^+ \bar{u}_f \left((y_u V_{CKM}) \hat{P}_L - (V_{CKM} y_d) \hat{P}_R \right) d_g \right]$$

$$+ \left[-(y_d)_{fg} \tfrac{1}{\sqrt{2}} v\, \bar{d}_f d_g - (y_d)_{fg} \tfrac{1}{\sqrt{2}} h\, \bar{d}_f d_g - i (y_d)_{fg} \tfrac{1}{\sqrt{2}} \phi_Z\, \bar{d}_f \gamma_5 d_g \right]$$

$$+ \left[\phi^- \bar{e}_f \left((y_\nu)_{fg} \hat{P}_R - (y_\ell)_{fg} \hat{P}_L \right) \nu_g + \phi^+ \bar{\nu}_f \left((y_\nu)_{fg} \hat{P}_L - (y_\ell)_{fg} \hat{P}_R \right) e_g \right]$$

$$+ \left[-(y_\ell)_{fg} \tfrac{1}{\sqrt{2}} v\, \bar{e}_f e_g - (y_\ell)_{fg} \tfrac{1}{\sqrt{2}} h\, \bar{e}_f e_g - i (y_\ell)_{fg} \tfrac{1}{\sqrt{2}} \phi_Z\, \bar{e}_f \gamma_5 e_g \right]$$

$\longrightarrow$ leads to mass terms, $\frac{y_{fg} v}{\sqrt{2}} = m_{fg}$

$\longrightarrow$ Higgs-fermion Yukawa interactions

$\longrightarrow$ Goldstone-fermion Yukawa interactions.

(in 4-component spinor notation)

$$\mathcal{L}_{Yuk} = -(m_U)_{fg}\, \bar{u}_f u_g - (m_D)_{fg}\, \bar{d}_f d_g - (m_\ell)_{fg}\, \bar{e}_f e_g$$

Quark/Lepton mass terms

$$- \frac{(y_u)_{fg}}{\sqrt{2}} h\, \bar{u}_f u_g - \frac{(y_d)_{fg}}{\sqrt{2}} h\, \bar{d}_f d_g - \frac{(y_\ell)_{fg}}{\sqrt{2}} h\, \bar{e}_f e_g$$

Yukawa couplings

$$+ \phi^+ \bar{u}_f \left((y_u V_{CKM})_{fg} \hat{P}_L - (V_{CKM} y_d)_{fg} \hat{P}_R \right) d_g$$

$$+ \phi^- \bar{d}_f \left((V_{CKM}^\dagger y_u)_{fg} \hat{P}_R - (y_d V_{CKM}^\dagger)_{fg} \hat{P}_L \right) u_g$$

$$+ \frac{i (y_u)_{fg}}{\sqrt{2}} \phi_Z\, \bar{u}_f \gamma_5 u_g - \frac{i (y_d)_{fg}}{\sqrt{2}} \phi_Z\, \bar{d}_f \gamma_5 d_f$$

Goldstone -Fermion couplings.

$$+ \phi^+ \bar{\nu}_f \left((y_\nu)_{fg} \hat{P}_L - (y_\ell) \hat{P}_R \right) e_g + \phi^- \bar{e}_f \left((y_\nu)_{fg} \hat{P}_R - (y_\ell)_{fg} \hat{P}_L \right) \nu_g$$

$$- \frac{i (y_\ell)_{fg}}{\sqrt{2}} \phi_Z\, \bar{e}_f \gamma_5 e_g$$