Neutrino masses & mixings

Quick way to include neutrino masses is by analogy with quark sector:

Add gauge singlet fermion: $\bar{\nu}_f$ $\quad f = \{1, 2, 3\}$.

Then,

$$\mathcal{L}_{Yuk.} = \ldots \underbrace{- H \cdot \ell_f^\dagger (Y_\ell)_{fg} \bar{e}_g^\dagger}_{\text{already in SM.}} - \tilde{H} \cdot \ell_f^\dagger (Y_\nu)_{fg} \bar{\nu}_g^\dagger + \text{h.c.}$$

$$= \ldots - \begin{pmatrix} \nu^\dagger \\ e^\dagger \end{pmatrix}_f \cdot (\phi^+, \phi^0) \begin{pmatrix} Y_\ell \end{pmatrix}_{fg} \bar{e}_g^\dagger + \text{h.c.}$$

$$- \begin{pmatrix} \nu^\dagger \\ e^\dagger \end{pmatrix}_f \cdot (\phi^{0*}, -\phi^-) \begin{pmatrix} Y_\nu \end{pmatrix}_{fg} \bar{\nu}_g^\dagger + \text{h.c.}$$

open up dot product:

$$= \left[-(Y_\ell)_{fg} \phi^+ \nu_f^\dagger \bar{e}_g^\dagger + \text{h.c.} \right] + \left[-(Y_\ell)_{fg} \phi^0 e_f^\dagger \bar{e}_g^\dagger + \text{h.c.} \right]$$

$$+ \left[-(Y_\nu)_{fg} \phi^{0*} \nu_f^\dagger \bar{\nu}_g^\dagger + \text{h.c.} \right] + \left[+(Y_\nu)_{fg} \phi^- e_f^\dagger \bar{\nu}_g^\dagger + \text{h.c.} \right]$$

Shift Higgs: $\phi^0 \to \frac{1}{\sqrt{2}} (v + h + i\phi_z)$

$\phi^{0*} \to \frac{1}{\sqrt{2}} (v + h - i\phi_z)$

explicitly write out h.c. terms:

Use: $(a_i \, Y_{ij} \, b_j)^\dagger = b_j^* (Y^\dagger)_{ji} \, a_i^* = b_i^* (Y^\dagger)_{ij} \, a_j^*$

$= (Y^\dagger)_{ij} \, b_i^* \, a_j^*$

$$= \left[-(y_\ell)_{fg}\, \phi^+\, \nu_f^\dagger \bar{e}_g^\dagger - (y_\ell^\dagger)_{fg}\, \phi^-\, \bar{e}_f\, \nu_g \right]$$

$$+ \left[-(y_\ell)_{fg}\, \frac{1}{\sqrt{2}}(v+h+i\phi_z)\, e_f^\dagger \bar{e}_g^\dagger - (y_\ell^\dagger)_{fg}\, \frac{1}{\sqrt{2}}(v+h-i\phi_z)\, \bar{e}_f\, e_g \right]$$

$$+ \left[-(y_\nu)_{fg}\, \frac{1}{\sqrt{2}}(v+h-i\phi_z)\, \nu_f^\dagger \bar{\nu}_g^\dagger - (y_\nu^\dagger)_{fg}\, \frac{1}{\sqrt{2}}(v+h+i\phi_z)\, \bar{\nu}_f\, \nu_g \right]$$

$$+ \left[+(y_\nu)_{fg}\, \phi^-\, e_f^\dagger \bar{\nu}_g^\dagger + (y_\nu^\dagger)_{fg}\, \phi^+\, \bar{\nu}_f\, e_g \right]$$

Convert to 4-component spinors

$$= \left[\phi^-\, \bar{e}_f \left((y_\nu)_{fg}\, \hat{P}_R - (y_\ell)_{fg}\, \hat{P}_L \right) \nu_g + \phi^+\, \bar{\nu}_f \left((y_\nu)_{fg}\, \hat{P}_L - (y_\ell)_{fg}\, \hat{P}_R \right) e_g \right]$$

$$+ \left[-(y_\nu)_{fg}\, \frac{1}{\sqrt{2}} v\, \bar{\nu}_f \nu_g - (y_\nu)_{fg}\, \frac{1}{\sqrt{2}} h\, \bar{\nu}_f \nu_g + i(y_\nu)_{fg}\, \frac{1}{\sqrt{2}} \phi_z\, \bar{\nu}_f \gamma_5 \nu_g \right]$$

$$+ \left[-(y_\ell)_{fg}\, \frac{1}{\sqrt{2}} v\, \bar{e}_f e_g - (y_\ell)_{fg}\, \frac{1}{\sqrt{2}} h\, \bar{e}_f e_g - i(y_e)_{fg}\, \frac{1}{\sqrt{2}} \phi_z\, \bar{e}_f \gamma_5 e_g \right]$$

Mass terms: $-(y_\nu)_{fg}\, \frac{1}{\sqrt{2}} v\, \bar{\nu}_f \nu_g$ (Neutrino mass) $- (y_\ell)_{fg}\, \frac{1}{\sqrt{2}} v\, \bar{e}_f e_g$ (charged lepton mass).