<u>Vanishing of the vacuum energy</u>

If SUSY is unbroken, then vacuum energy density vanishes:

<u>PROOF</u>

Start with SUSY algebra

$$\{\hat{Q}_\alpha, \hat{\bar{Q}}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

$$\sigma^0 = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

Set $\dot\beta = \alpha$ and sum over α.

$$\sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$\{\hat{Q}_1, \hat{\bar{Q}}_{\dot 1}\} + \{Q_2, \hat{\bar{Q}}_{\dot 2}\} = 2\left(\sigma^\mu_{1\dot 1}\hat{P}_\mu + \sigma^\mu_{2\dot 2}\hat{P}_\mu\right)$$

In RHS only σ^0 & σ^3 are diagonal.

$$= 2\left(\sigma^0_{1\dot 1}\hat{P}_0 + \sigma^0_{2\dot 2}\hat{P}_0 + \sigma^3_{1\dot 1}\hat{P}_3 + \sigma^3_{2\dot 2}\hat{P}_3\right)$$

$$= 2\left(\hat{P}_0 + \hat{P}_0 + \cancel{P_0} - \cancel{P_0}\right)$$

$$= 4\hat{H}$$

$$\hat{H} = \frac{1}{4}\left[\hat{Q}_1\hat{\bar{Q}}_{\dot 1} + \hat{\bar{Q}}_{\dot 1}\hat{Q}_1 + \hat{Q}_2\hat{\bar{Q}}_{\dot 2} + \hat{\bar{Q}}_{\dot 2}\hat{Q}_2\right]$$

sandwich both sides with vacuum state

$$\langle\Omega|\hat{H}|\Omega\rangle = \frac{1}{4}\langle\Omega|\,\hat{Q}_1\hat{\bar{Q}}_{\dot 1} + \hat{\bar{Q}}_{\dot 1}\hat{Q}_1 + \hat{Q}_2\hat{\bar{Q}}_{\dot 2} + \hat{\bar{Q}}_{\dot 2}\hat{Q}_2\,|\Omega\rangle$$

The definition of unbroken SUSY is:

$$\hat{Q}_{1\text{ or }2}|\Omega\rangle = \hat{\bar{Q}}_{1\text{ or }2}|\Omega\rangle = 0.$$

$$\therefore \quad \boxed{\langle\Omega|\hat{H}|\Omega\rangle = 0}$$