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Covariant Derivatives

To construct Lagrangians in superspace,
need to be able to take derivatives with respect to θ_α and $\theta^\dagger_{\dot\alpha}$

But, $\dfrac{\partial}{\partial\theta^\alpha}$ and $\dfrac{\partial}{\partial\theta^\dagger_{\dot\alpha}}$ are not SUSY covariant.

i.e. $\left[Q, \dfrac{\partial}{\partial\theta^\alpha}\right] \neq 0.$

The appropriate covariant derivatives are the ones found during the verification of SUSY algebra $\{Q, Q^\dagger\}$.

$$\boxed{\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\beta}\theta^{\dagger\dot\beta}\partial_\mu & \qquad D^\alpha &= \frac{-\partial}{\partial\theta_\alpha} + i(\theta^\dagger_{\dot\beta}\bar\sigma^{\mu\dot\beta\alpha})\partial_\mu \\ D^\dagger_{\dot\alpha} &= \frac{-\partial}{\partial\theta^{\dagger\dot\alpha}} + i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu & \qquad D^{\dagger\dot\alpha} &= \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}} - i(\bar\sigma^{\mu\dot\alpha\beta}\theta_\beta)\partial_\mu \end{aligned}}$$

- Satisfy the following anticommutation relations

$$\{D_\alpha, Q_\beta\} = \{D_\alpha, Q^\dagger_{\dot\beta}\} = 0$$

$$\{D^\dagger_{\dot\alpha}, Q_\beta\} = \{D^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0$$

D and $D^\dagger$ are SUSY covariant:
acting on superfield gives back another superfield

Note also:

$$\{D_\alpha, D_\beta\} = \{D^\dagger_{\dot\alpha}, D^\dagger_{\dot\beta}\} = 0$$

but $\{D_\alpha, D^\dagger_{\dot\beta}\} = -2\sigma^\mu_{\alpha\dot\beta}\, i\partial_\mu$ c.f. $\{Q_\alpha, Q^\dagger_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta}P_\mu$

$$= -2\sigma^\mu_{\alpha\dot\beta}P_\mu$$

- D and $D^\dagger$ satisfy a supersymmetry algebra with the wrong sign.
(The non-vanishing [anti] commutator implies that even global superspace has non-vanishing curvature ~ torsion)!
- The D and $D^\dagger$ may be viewed as the covariant "square-root" of ∂_μ.

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Also useful:

$$\int d^2\theta \; D_\alpha(\text{anything}) = \int d^2\theta \left[\underbrace{\frac{\partial}{\partial\theta^\alpha}}_{\text{vanishes.}} - i(\sigma^\mu\theta^\dagger)_\alpha \partial_\mu \right](\text{anything})$$

$$= \partial_\mu \left[-i(\sigma^\mu\theta^\dagger)_\alpha \int d^2\theta \, (\text{anything}) \right] = \text{total derivative in } x^\mu.$$

$$\int d^2\theta^\dagger \; D^\dagger_{\dot\alpha}(\text{anything}) = \partial_\mu \left[i(\theta\sigma^\mu)_{\dot\alpha} \int d^2\theta^\dagger (\text{anything}) \right] = \text{total derivative in } x^\mu.$$