Vector superfields

$\boxed{\text{Satisfies the constraint } V = V^*}$ $\quad {}^* \equiv$ complex conjugate

$$\Sigma(x,\theta,\theta^\dagger) = \phi + \chi^\alpha\theta_\alpha + \theta^\dagger_{\dot\alpha}\xi^{\dagger\dot\alpha} + \theta^\alpha\theta_\alpha f + \theta^\dagger_{\dot\alpha}\theta^{\dagger\dot\alpha} g + \theta^\alpha\sigma^\mu_{\alpha\dot\beta}\theta^{\dagger\dot\beta} V_\mu$$
$$+ \theta^\dagger_{\dot\alpha}\zeta^{\dagger\dot\alpha}\theta^\beta\theta_\beta + \theta^\dagger_{\dot\alpha}\theta^{\dagger\dot\alpha}\eta^\beta\theta_\beta + \theta^\dagger_{\dot\alpha}\theta^{\dagger\dot\alpha}\theta^\beta\theta_\beta\, d(x).$$

$$\Sigma^*(x,\theta,\theta^\dagger) = \phi^* + \theta^\dagger_{\dot\alpha}\chi^{\dagger\dot\alpha} + \xi^\alpha\theta_\alpha + \theta^\dagger_{\dot\alpha}\theta^{\dagger\dot\alpha} f^* + \theta^\alpha\theta_\alpha g^* + \theta^\alpha\sigma^\mu_{\alpha\dot\beta}\theta^{\dagger\dot\beta} V^*_\mu$$
$$+ \theta^\dagger_{\dot\alpha}\theta^{\dagger\dot\alpha}\zeta^\beta\theta_\beta + \theta^\alpha\theta_\alpha\theta^\dagger_{\dot\beta}\eta^{\dagger\dot\beta} + \theta^\dagger_{\dot\alpha}\theta^{\dagger\dot\alpha}\theta^\beta\theta_\beta\, d^*(x)$$

match:

$\phi(x) = \phi^*(x)$ Reals $\qquad V_\mu(x) = V^*_\mu(x)$ Reals

$\xi^\dagger_{\dot\alpha}(x) = \chi^\dagger_{\dot\alpha}(x) \qquad \zeta^\dagger_{\dot\alpha} = \eta^\dagger_{\dot\alpha}$

$f(x) = g(x) \qquad d(x) = d^*(x)$ Reals.

then:

$$V(x,\theta,\theta^\dagger) = \phi(x) + \chi(x)\theta + \theta^\dagger\chi^\dagger(x) + \theta\theta f(x) + \theta^\dagger\theta^\dagger f^*(x)$$
$$+ \theta\sigma^\mu\theta^\dagger V_\mu(x) + \theta^\dagger\eta^\dagger(x)\theta\theta + \theta^\dagger\theta^\dagger\eta(x)\theta + \theta^\dagger\theta^\dagger\theta\theta\, d(x)$$

reason: Makes λ and D transform homogeneously under gauge transformations

Convenient to define

$$\eta_\alpha = \lambda_\alpha - \frac{i}{2}(\sigma^\mu\partial_\mu\chi^\dagger)_\alpha \Rightarrow \eta^{\dagger\dot\alpha} = \lambda^{\dagger\dot\alpha} - \frac{i}{2}(\bar\sigma^\mu\partial_\mu\chi)^{\dot\alpha}$$

$$V_\mu = A_\mu \quad \text{and} \quad d(x) = \frac{1}{2}D(x) - \frac{1}{4}\partial^2\phi(x)$$

$\Rightarrow$

$$\boxed{\begin{aligned} V(x,\theta,\theta^\dagger) = {}& \phi(x) + \chi(x)\theta + \theta^\dagger\chi^\dagger(x) + \theta\theta f(x) + \theta^\dagger\theta^\dagger f^*(x) \\ & + \theta\sigma^\mu\theta^\dagger A_\mu(x) + \theta^\dagger\left(\lambda^\dagger(x) - \frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right)\theta\theta \\ & + \theta^\dagger\theta^\dagger\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\chi^\dagger(x)\right) + \theta^\dagger\theta^\dagger\theta\theta\left[\frac{1}{2}D(x) - \frac{1}{4}\partial^2\phi(x)\right] \end{aligned}}$$

Component transformation rule for vector superfield.

$$\delta\phi = \alpha\chi + \alpha^\dagger\chi^\dagger$$

$$\delta\chi = 2\alpha f + \sigma^\mu\alpha^\dagger A_\mu - i\sigma^\mu\alpha^\dagger\partial_\mu\phi = 2\alpha f + \sigma^\mu\alpha^\dagger(A_\mu - i\partial_\mu\phi)$$

$$\delta\xi^\dagger = \delta\chi^\dagger = 2\alpha^\dagger f^* + \bar\sigma^\mu\alpha A_\mu - i\bar\sigma^\mu\alpha\,\partial_\mu\phi = (\delta\chi)^\dagger$$ consistency ✓

$$\delta f = \alpha^\dagger(\lambda^\dagger - \tfrac{i}{2}\bar\sigma^\mu\partial_\mu\chi) - \tfrac{i}{2}\alpha^\dagger\bar\sigma^\mu\partial_\mu\chi$$

$$= \alpha^\dagger\lambda^\dagger - i\alpha^\dagger\bar\sigma^\mu\partial_\mu\chi$$

$$\delta g = \delta f^* = \alpha(\lambda - \tfrac{i}{2}\sigma^\mu\partial_\mu\chi^\dagger) - \tfrac{i}{2}\alpha\sigma^\mu\partial_\mu\chi^\dagger$$

$$= \alpha\lambda - i\alpha\sigma^\mu\partial_\mu\chi^\dagger = (\delta f)^*$$ constent ✓

$$\delta A_\mu = \alpha\sigma_\mu(\lambda^\dagger - \tfrac{i}{2}\bar\sigma_\nu\partial^\nu\chi) - \alpha^\dagger\bar\sigma_\mu(\lambda - \tfrac{i}{2}\sigma_\nu\partial^\nu\chi^\dagger) - \tfrac{i}{2}\alpha\sigma_\nu\bar\sigma_\mu\partial^\nu\chi + \tfrac{i}{2}\alpha^\dagger\bar\sigma_\nu\sigma_\mu\partial^\nu\chi^\dagger$$

$$\cancel{= \alpha\sigma_\mu\lambda^\dagger - \alpha^\dagger\bar\sigma_\mu\lambda - \tfrac{i}{2}\alpha\sigma_\mu\bar\sigma_\nu\partial^\nu\chi + \tfrac{i}{2}\alpha^\dagger\bar\sigma_\mu\sigma_\nu\partial^\nu\chi^\dagger + (\quad \text{same} \quad)}$$

$$= \alpha\sigma_\mu\lambda^\dagger - \alpha^\dagger\bar\sigma_\mu\lambda - \tfrac{i}{2}\alpha\underbrace{(\sigma_\mu\bar\sigma_\nu + \sigma_\nu\bar\sigma_\mu)}_{2g_{\mu\nu}}\partial^\nu\chi + \tfrac{i}{2}\alpha^\dagger\underbrace{(\bar\sigma_\mu\sigma_\nu + \bar\sigma_\nu\sigma_\mu)}_{2g_{\mu\nu}}\partial^\nu\chi^\dagger$$

$$= \alpha\sigma_\mu\lambda^\dagger - \alpha^\dagger\bar\sigma_\mu\lambda - i\alpha\partial_\mu\chi + i\alpha^\dagger\partial_\mu\chi^\dagger$$

$$\delta\zeta^\dagger = \delta(\lambda^\dagger - \tfrac{i}{2}\bar\sigma^\mu\partial_\mu\chi) = \delta\lambda^\dagger - \tfrac{i}{2}\bar\sigma^\mu\partial_\mu\underbrace{\delta\chi}_{\text{known from above}} = \delta\lambda^\dagger - \tfrac{i}{2}\bar\sigma^\mu\partial_\mu(2\alpha f + \sigma^\nu\alpha^\dagger A_\nu - i\sigma^\nu\alpha^\dagger\partial_\nu\phi)$$

$$= \delta\lambda^\dagger - \tfrac{i}{2}\bar\sigma^\mu\alpha\partial_\mu f - \tfrac{i}{2}\bar\sigma^\mu\sigma^\nu\alpha^\dagger\partial_\mu A_\nu - \tfrac{1}{2}\underbrace{\bar\sigma^\mu\sigma^\nu}_{g^{\mu\nu}}\alpha^\dagger\partial_\mu\partial_\nu\phi$$

$$= 2\alpha^\dagger\left[\tfrac{1}{2}D - \tfrac{1}{4}\partial^2\phi\right] - i\bar\sigma^\mu\alpha\partial_\mu f - \tfrac{i}{2}\bar\sigma^\nu\sigma^\mu\alpha^\dagger\partial_\mu A_\nu$$

$$= \alpha^\dagger D - \tfrac{1}{2}\alpha^\dagger\partial^2\phi - i\bar\sigma^\mu\alpha\partial_\mu f - \tfrac{i}{2}\bar\sigma^\nu\sigma^\mu\alpha^\dagger\partial_\mu A_\nu$$

$$= \alpha^\dagger D \underbrace{- \tfrac{1}{2}\alpha^\dagger\partial^2\phi - i\bar\sigma^\mu\alpha\partial_\mu f - \tfrac{i}{2}\bar\sigma^\mu\sigma^\nu\alpha^\dagger\partial_\mu A_\nu}_{-\frac{i}{2}\bar\sigma^\mu\partial_\mu\delta\chi} + \tfrac{i}{2}\underbrace{(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu)}_{\to 2\bar\sigma^\mu\sigma^\nu}\underbrace{\alpha^\dagger\partial_\mu A_\nu}_{\frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)}$$

$$\Rightarrow \quad \delta\lambda^\dagger = \alpha^\dagger D + \tfrac{i}{2}\bar\sigma^\mu\sigma^\nu(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

$$\delta\eta \to \quad \delta\lambda = \alpha D - \tfrac{i}{2}\sigma^\mu\bar\sigma^\nu(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

$$\delta d = \delta\left[\tfrac{1}{2}D - \tfrac{1}{4}\partial^2\phi\right] = \tfrac{1}{2}\delta D - \tfrac{1}{4}\partial^2\delta\phi$$

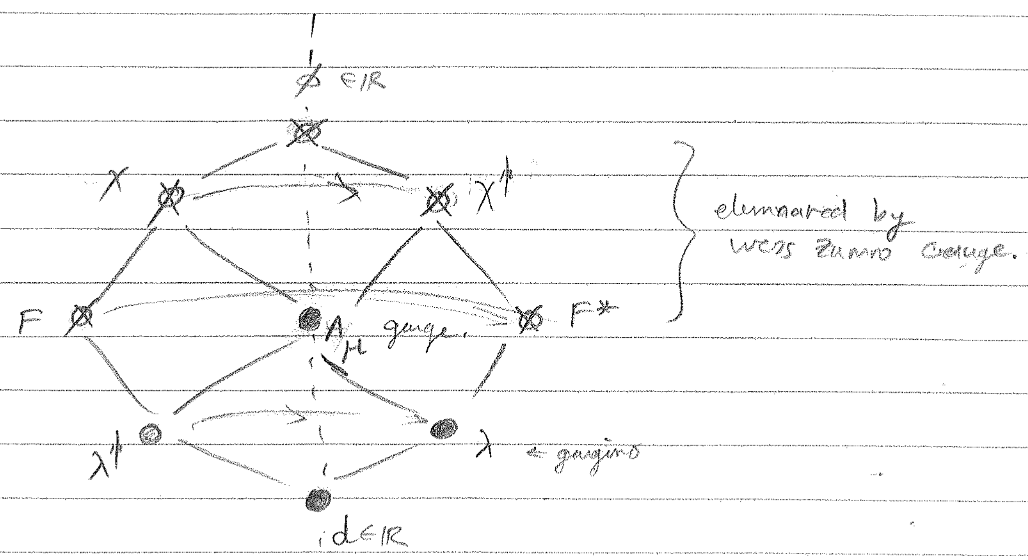
$$= \tfrac{1}{2}\delta D - \tfrac{1}{4}\partial^2\alpha\chi - \tfrac{1}{4}\partial^2\alpha^\dagger\chi^\dagger$$

$$= -\tfrac{i}{2}\alpha\sigma^\mu\partial_\mu(\lambda^\dagger - \tfrac{i}{2}\bar\sigma^\nu\partial_\nu\chi) - \tfrac{i}{2}\alpha^\dagger\bar\sigma^\mu\partial_\mu(\lambda - \tfrac{i}{2}\sigma^\nu\partial_\nu\chi^\dagger)$$

$$= -\tfrac{i}{2}\alpha\sigma^\mu\partial_\mu\lambda^\dagger - \tfrac{i}{2}\alpha^\dagger\bar\sigma^\mu\partial_\mu\lambda - \tfrac{1}{4}\alpha\underbrace{\sigma^\mu\bar\sigma^\nu}_{g^{\mu\nu}}\partial_\mu\partial_\nu\chi - \tfrac{1}{4}\alpha^\dagger\underbrace{\bar\sigma^\mu\sigma^\nu}_{g^{\mu\nu}}\partial_\mu\partial_\nu\chi^\dagger$$

$$= -\tfrac{i}{2}\alpha\sigma^\mu\partial_\mu\lambda^\dagger - \tfrac{i}{2}\alpha^\dagger\bar\sigma^\mu\partial_\mu\lambda - \tfrac{1}{4}\partial^2\alpha\chi - \tfrac{1}{4}\partial^2\alpha^\dagger\chi^\dagger.$$

$\Rightarrow \delta D = -i\alpha\sigma^\mu\partial_\mu\lambda^\dagger - i\alpha^\dagger\bar\sigma^\mu\partial_\mu\lambda$ (still) a total derivative.



Important to note: