Gauge transformation of vector superfield:

Let: $V(x, \theta, \theta^\dagger)$ be a vector superfield for $U(1)$ gauge symmetry.

Consider the transformation:

$$V \longrightarrow V - i(\Omega - \Omega^*)$$

where Ω = space-time dependent chiral superfield gauge transformation parameter:

$$\Omega = \beta + \theta\sqrt{2}\omega + \theta\theta s + \theta\sigma^\mu\theta^\dagger(-i\partial_\mu\beta) + \theta^\dagger\left[\frac{-i}{\sqrt{2}}\bar{\sigma}^\mu\partial_\mu\omega\right]\theta\theta + \theta^\dagger\theta^\dagger\theta\theta\left[\frac{-1}{4}\partial^2\beta\right]$$

$$\Omega^* = \beta^* + \sqrt{2}\omega^\dagger\theta^\dagger + \theta^\dagger\theta^\dagger s^* + \theta\sigma^\mu\theta^\dagger(i\partial_\mu\beta^*) + \theta^\dagger\theta^\dagger\left[\frac{i}{\sqrt{2}}\partial_\mu\omega^\dagger\bar{\sigma}^\mu\right]\theta + \theta^\dagger\theta^\dagger\theta\theta\left[\frac{1}{4}\partial^2\beta^*\right]$$

so that, in components

$$\phi \longrightarrow \phi - i(\beta - \beta^*)$$

$$\chi \longrightarrow \chi + \sqrt{2}i\omega \qquad \chi^\dagger \longrightarrow \chi^\dagger - 2i\omega^\dagger$$

$$f \rightarrow f + s \qquad f^* \longrightarrow f^* + s^*$$

$$A_\mu \longrightarrow A_\mu - i\partial_\mu(\beta + \beta^*)$$

$$\left(\lambda^\dagger - \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi\right) \longrightarrow \lambda^\dagger \underbrace{- \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi + \frac{1}{\sqrt{2}}\bar{\sigma}^\mu\partial_\mu\omega}$$

cancel (see above)

$$\Rightarrow \lambda^\dagger \longrightarrow \lambda^\dagger \quad \text{(invariant, by design)}$$

$$\lambda \longrightarrow \lambda \quad (\text{''} , \text{''} \quad \text{''})$$

$$\frac{1}{2}D - \frac{1}{4}\partial^2\phi \longrightarrow \frac{1}{2}D \underbrace{- \frac{1}{4}\partial^2\phi + \frac{i}{4}\partial^2(\beta^* - \beta)}$$

cancel (see above)

$$D \longrightarrow D \quad \text{(invariant, by design)}$$

Super-gauge transformation parameter represents 8 degrees of freedom:

$$\mathrm{Re}(\beta),\quad \underbrace{\mathrm{Im}(\beta),\quad \omega,\quad s}_{\text{use these to eliminate}}$$

$$1 \quad + \quad 1 \quad + \quad 4 \quad + \quad 2 \quad = 2$$

use these to eliminate

ϕ, χ, f in vector superfield:

$$\boxed{V_{W.Z.}(x;\theta,\theta^\dagger) = \theta\sigma^\mu\theta^\dagger A_\mu(x) + \theta^\dagger\lambda^\dagger(x)\theta\theta + \theta^\dagger\theta^\dagger\theta\lambda(x) + \tfrac{1}{2}\theta^\dagger\theta^\dagger\theta\theta D(x).}$$

leaving $A_\mu \to A_\mu - i\partial_\mu 2\,\mathrm{Re}[\beta]$ as the residual gauge transformation.

Notes:

- Under a SUSY transformation, the vector superfield no longer satisfies Wess-Zumino gauge.

$\Rightarrow$ Wess-Zumino gauge is not a SUSY-invariant gauge.
(like how Coulomb or Axial gauges are not Lorentz invariant).

- In terms of $y^\mu = x^\mu - i\theta\sigma^\mu\theta^\dagger$, vector superfield in the Wess-Zumino gauge is not very different:

$$A_\mu(x) = A_\mu(y + i\theta\sigma^\mu\theta^\dagger)$$ (all other components give higher order in θ & $\theta^\dagger$

and use $(\theta\sigma^\mu\theta^\dagger)(\theta\sigma^\nu\theta^\dagger) = \frac{1}{2}g^{\mu\nu}(\theta\theta)(\theta^\dagger\theta^\dagger)$ calculated previously:

$$V_{W.Z.}(y;\theta,\theta^\dagger) = \theta\sigma^\mu\theta^\dagger A_\mu(y) + \theta^\dagger\lambda^\dagger(y)\theta\theta + \theta^\dagger\theta^\dagger\theta\lambda(y) + \frac{1}{2}\theta^\dagger\theta^\dagger\theta\theta\left[D(y) + i\partial\cdot A(y)\right]$$

Properties of Vector superfields

Vector superfields may be written in terms of chiral superfields.

Obvious if I write

"Real" " " " " " " " " "Complex" "

Φ = left-chiral superfield

Φ^* = right-chiral superfield

Then $\underbrace{\Phi + \Phi^*}_{\text{"Real part"}}$, $\underbrace{i(\Phi - \Phi^*)}_{\text{"Imaginary part"}}$, $\underbrace{\Phi\Phi^*}_{\text{"Modulus"}}$ are vector superfields.