Building a supersymmetric Lagrangian

- Two types of terms may enter into the action:
  (named after components of the superfield transforming as a total derivative)

F-term:

Φ = left-chiral

$$[\Phi]_F = \int d^2\theta\, \Phi\big|_{\theta^\dagger=0} = F(x) \xrightarrow{\text{SUSY}} -i\sqrt{2}\,\alpha^\dagger\bar{\sigma}^\mu\partial_\mu\chi(x)$$

c.c.

Φ^* = right-chiral

$$[\Phi^*]_F = \int d^2\theta^\dagger\, \Phi^*\big|_{\theta=0} = F^*(x) \longrightarrow i\sqrt{2}\,\partial_\mu\chi^\dagger(x)\,\bar{\sigma}^\mu\alpha$$

(Total derivative)

Since the action is real (to generate a Hermitian Hamiltonian), both terms must be included:

$$S = \int d^4x \left([\Phi]_F + \text{c.c.}\right) \qquad (*)$$

D-term:

$$[V]_D = \int d^2\theta\, d^2\theta^\dagger\, V = D(x) \xrightarrow{\text{SUSY}} -i\left(\alpha\sigma^\mu\partial_\mu\lambda^\dagger + \alpha^\dagger\bar{\sigma}^\mu\partial_\mu\lambda\right)$$

Since the vector superfield is real (by construction) D-terms can be added as is:

$$S = \int d^4x\, [V]_D \qquad (**)$$

Contributions (*) and (**) are the only types of terms that may be added to the action in a (four-dimensional) supersymmetric theory.