Start with a <u>chiral superfield</u>, Φ.

$\boxed{\text{Try to write D-terms:}}$

(First, must form vector fields; then extract D-terms)

$$\underbrace{[\Phi + \Phi^*]_D \qquad [i(\Phi - \Phi^*)]_D}_{\substack{\text{these terms are typically not gauge-inv,} \\ \text{and at any rate, are total divergences } \sim \partial^2\phi}} \qquad \underset{\substack{\downarrow \\ \text{OK} \\ \text{(Gives kinetic terms)}}}{[\Phi^*\Phi]_D} \qquad + \underset{\substack{\downarrow \\ \text{lead to higher} \\ \text{dimensional (non-renorm.)}}}{\left[\substack{\text{products} \\ \text{of these}}\right]_D}$$

Left & right chiral superfields:

$$\Phi^* = \phi^* + i\theta\sigma^\mu\theta^\dagger\partial_\mu\phi^* - \frac{1}{4}\theta\theta\theta^\dagger\theta^\dagger\partial^2\phi^* + \sqrt{2}\,\theta^\dagger\chi^\dagger + \frac{i}{\sqrt{2}}\theta^\dagger\theta^\dagger\partial_\mu\chi^\dagger\bar\sigma^\mu\theta + \theta^\dagger\theta^\dagger F^*$$

$$\Phi = \phi - i\theta\sigma^\mu\theta^\dagger\partial_\mu\phi - \frac{1}{4}\theta\theta\theta^\dagger\theta^\dagger\partial^2\phi + \sqrt{2}\,\theta\chi - \frac{i}{\sqrt{2}}\theta\theta\theta^\dagger\bar\sigma^\mu\partial_\mu\chi + \theta\theta F$$

So

$$\Phi^*\Phi = \ldots\ \phi^*\left(\frac{-1}{4}\theta\theta\theta^\dagger\theta^\dagger\partial^2\phi\right) + \overset{(A)}{\theta\sigma^\mu\theta^\dagger\theta\sigma^\nu\theta^\dagger}\partial_\mu\phi^*\partial_\nu\phi + \left(\frac{-1}{4}\theta\theta\theta^\dagger\theta^\dagger\right)(\partial^2\phi^*)\phi$$

$$\overset{(B)}{+\sqrt{2}(\theta^\dagger\chi^\dagger)\left(\frac{-i}{\sqrt{2}}\theta\theta\theta^\dagger\bar\sigma^\mu\partial_\mu\chi\right)} + \overset{(C)}{\frac{i}{\sqrt{2}}\theta^\dagger\theta^\dagger\partial_\mu\chi^\dagger\bar\sigma^\mu\theta\sqrt{2}(\theta\chi)}$$

$$+\theta^\dagger\theta^\dagger F^*\theta\theta F$$

Now use (A): $(\theta\sigma^\mu\theta^\dagger)(\theta\sigma^\nu\theta^\dagger) = \frac{1}{2}g^{\mu\nu}(\theta\theta)(\theta^\dagger\theta^\dagger)$

(B) $\theta^\dagger_{\dot\alpha}\chi^{\dagger\dot\alpha}\theta^\dagger_{\dot\beta}\bar\sigma^{\mu\dot\beta\gamma}\partial_\mu\chi_\gamma$

$$= \frac{1}{2}\epsilon_{\dot\beta\dot\alpha}(\theta^\dagger\theta^\dagger)\chi^{\dagger\dot\alpha}\bar\sigma^{\mu\dot\beta\gamma}\partial_\mu\chi_\gamma$$

$$= +\frac{1}{2}(\theta^\dagger\theta^\dagger)\chi^{\dagger\dot\beta}\bar\sigma^{\mu\dot\beta\gamma}\partial_\mu\chi_\gamma$$

(C) $\partial_\mu\chi^\dagger_{\dot\alpha}\bar\sigma^{\mu\dot\alpha\beta}\theta_\beta\theta^\gamma\chi_\gamma$

$$= \partial_\mu\chi^\dagger_{\dot\alpha}\bar\sigma^{\mu\dot\alpha\beta}\underbrace{\theta_\beta(-\theta_\gamma}_{\frac{1}{2}\epsilon_{\beta\gamma}(\theta\theta)}\chi^\gamma)$$

$$= -\frac{1}{2}\partial_\mu\chi^\dagger\bar\sigma^{\mu\dot\alpha\beta}(\theta\theta)\chi_\beta$$

$$\Phi^* \Phi = (\theta\theta\,\theta^\dagger\theta^\dagger)\Big[-\frac{1}{4}\phi^* \partial^2 \phi + \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi - \frac{1}{4}(\partial^2\phi)^* \phi + \frac{i}{2}\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi - \frac{i}{2}\partial_\mu \chi^\dagger \bar{\sigma}^\mu \chi + F^* F \Big]$$

Then D-term becomes:

$$\int d^4x\, [\Phi^*\Phi]_D = \int d^4x \Big(-\frac{1}{4}\phi^* \partial^2 \phi + \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi - \frac{1}{4}(\partial^2\phi)^* \phi + \frac{i}{2}\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi - \frac{i}{2}\partial_\mu \chi^\dagger \bar{\sigma}^\mu \chi + F^* F \Big)$$

Integrate by parts on first & third terms $\rightarrow$ $\partial_\mu \phi^* \partial^\mu \phi$

on fifth term $\longrightarrow$ $i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi$

$$\int d^4x\, [\Phi^*\Phi]_D = \int d^4x \Big(\partial_\mu \phi^* \partial^\mu \phi + i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi + F^* F \Big)$$