Now, vector superfield:

Need field strength "tensor" to provide vector superfield with dynamics.

really a spinor.

Requirements for W_α

- Must satisfy chirality: $D^\dagger_{\dot\alpha} W_\beta = 0$
- Transforms gauge-covariantly $\Rightarrow$ no inhomogeneous part
  [implies invariance for U(1) gauge group]

By trial-and-error, find [for U(1)]

$$W_\alpha = -\frac{1}{4}(D^\dagger D^\dagger) D_\alpha V \qquad \text{left-chiral (anticommuting)}$$

$$W^\dagger_{\dot\alpha} = -\frac{1}{4}(DD) D^\dagger_{\dot\alpha} V \qquad \text{right-chiral}$$

V = vector superfield (any gauge)

check invariance under U(1) gauge transformation:

Ω = gauge parameter. chiral superfield

$$W_\alpha \to -\frac{1}{4}(D^\dagger D^\dagger) D_\alpha \left(V - i(\Omega - \Omega^*)\right)$$

$$= -\frac{1}{4}(D^\dagger D^\dagger) D_\alpha V + \frac{i}{4}(D^\dagger D^\dagger) D_\alpha (\Omega - \Omega^*)$$

$D_\alpha \Omega^* = 0$ because Ω^* is right-chiral

$$= W_\alpha + \frac{i}{4}\underbrace{(D^\dagger D^\dagger) D_\alpha}_{\text{replace with (anti) commutator}} \Omega$$

$$= W_\alpha - \frac{i}{4} D^{\dagger\dot\beta} \{D^\dagger_{\dot\beta}, D_\alpha\} \Omega$$

unnatural index contraction!

ok because $D^\dagger_{\dot\alpha} \Omega = 0$

$$= W_\alpha - \frac{i}{4} D^{\dagger\dot\beta} (2\sigma^\mu_{\alpha\dot\beta} \partial_\mu) \Omega$$

commute $D^\dagger$ past ∂ $\Leftarrow$ $[D^\dagger, \partial] = 0$ by inspection

$$= W_\alpha - \frac{i}{4} 2\partial_\mu \sigma^\mu_{\alpha\dot\beta} \underbrace{D^{\dagger\dot\beta}\Omega}_{0}$$

$$= W_\alpha. \qquad \text{(ok).}$$

$W_\alpha = \frac{-1}{4}(D^\dagger D^\dagger) D_\alpha V$

Expressing W_α in terms of component fields

- For easier differentiation, write V in terms of $y, \theta, \theta^\dagger$ coordinates (recall $D^\dagger \sim \frac{\partial}{\partial\theta^\dagger}$)
- Since W_α is gauge inv. choose Wess-Zumino gauge for V.

$$V_{W.Z.}(y,\theta,\theta^\dagger) = \theta\sigma^\mu\theta^\dagger A_\mu(y) + \theta^\dagger\theta^\dagger\theta\lambda(y) + \theta\theta\theta^\dagger\lambda^\dagger(y) + \frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger\left[D(y) + i\partial_\mu A^\mu(y)\right]$$

Start by applying $D_\alpha = \frac{\partial}{\partial\theta^\alpha} - 2i(\sigma^\nu\theta^\dagger)_\alpha \frac{\partial}{\partial y^\nu}$:

$$D_\alpha V_{W.Z} = (\sigma^\mu\theta^\dagger)_\alpha A_\mu + \underset{②}{\theta^\dagger\theta^\dagger\lambda_\alpha} + 2\theta_\alpha\theta^\dagger\lambda^\dagger + \underset{④}{\theta_\alpha(\theta^\dagger\theta^\dagger)\left[D + i\partial_\mu A^\mu\right]}$$
$$\quad - \underset{⑤}{2i(\sigma^\nu\theta^\dagger)_\alpha(\theta\sigma^\mu\theta^\dagger)\partial_\nu A_\mu} + 0 - \underset{⑦}{2i(\sigma^\nu\theta^\dagger)_\alpha(\theta\theta)(\theta^\dagger\partial_\nu\lambda^\dagger)} + 0$$

Now apply $(D^\dagger_{\dot\alpha} D^{\dagger\dot\alpha}) = \frac{-\partial}{\partial\theta^{\dagger\dot\alpha}}\frac{\partial}{\partial\theta^\dagger_{\dot\alpha}} = \frac{+\partial}{\partial\theta^\dagger_{\dot\alpha}}\frac{\partial}{\partial\theta^{\dagger\dot\alpha}}$

Notes:

- First and third terms double-differentiate to zero.
- Identities:

For 2nd and 4th terms:

$$\frac{\partial}{\partial\theta^{\dagger\dot\alpha}}(\theta^\dagger_{\dot\beta}\theta^{\dagger\dot\beta}) = -2\theta^\dagger_{\dot\alpha}$$

② & ④ $\Rightarrow \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}}\frac{\partial}{\partial\theta^{\dagger\dot\alpha}}(\theta^\dagger\theta^\dagger) = -2\delta_{\dot\alpha}{}^{\dot\alpha} = -4.$

For last terms: change index $\alpha \to b$

(5) $\frac{\partial}{\partial\theta^{\dagger\dot\alpha}}\left[+2i(\sigma^\nu\theta^\dagger)_\alpha(\theta^\dagger\bar\sigma^\mu\theta)\right]$

$$= \frac{\partial}{\partial\theta^{\dagger\dot\alpha}}\left[+2i\,\sigma^\nu_{b\dot c}\,\theta^{\dagger\dot c}\,\theta^\dagger_{\dot d}\,\bar\sigma^{\dot d e\mu}\theta_e\right]$$

$$= 2i\,\sigma^\nu_{b\dot c}\,\delta^{\dot c}{}_{\dot\alpha}\,\theta^\dagger_{\dot d}\,\bar\sigma^{\dot d e\mu}\theta_e - 2i\,\sigma^\nu_{b\dot c}\,\theta^{\dagger\dot c}\,\epsilon_{\dot d\dot\alpha}\,\bar\sigma^{\dot d e\mu}\theta_e$$

$$= 2i\,\sigma^\nu_{b\dot\alpha}\,\theta^\dagger_{\dot d}\,\bar\sigma^{\dot d e\mu}\theta_e - 2i\,\sigma^\nu_{b\dot c}\,\theta^{\dagger\dot c}\,\epsilon_{\dot d\dot\alpha}\,\bar\sigma^{\dot d e\mu}\theta_e$$

then $\frac{\partial}{\partial\theta^\dagger_{\dot\alpha}}[\text{this}] = 2i\,\sigma^\nu_{b\dot\alpha}\,\delta^{\dot\alpha}{}_{\dot d}\,\bar\sigma^{\dot d e\mu}\theta_e + \sigma^\nu_{b\dot c}\,\underbrace{\epsilon^{\dot c\dot\alpha}\,\epsilon_{\dot d\dot\alpha}}_{-\delta^{\dot c}{}_{\dot d}}\,\bar\sigma^{\dot d e\mu}\theta_e$

$$= 2i\,\sigma^\nu_{b\dot d}\,\bar\sigma^{\dot d e\mu}\theta_e + 2i\,\sigma^\nu_{b\dot d}\,\bar\sigma^{\dot d e\mu}\theta_e$$

$$= 4i\,(\sigma^\nu\bar\sigma^\mu\theta)_\alpha$$ change $b\to\alpha$.

(7) $\frac{\partial}{\partial\theta^{\dagger\dot\alpha}}\left[-2i\,\sigma^\nu_{b\dot c}\,\theta^{\dagger\dot c}\,(\theta\theta)\,\theta^\dagger_{\dot e}\,\partial_\nu\lambda^{\dagger\dot e}\right]$

$$= -2i\,\sigma^\nu_{b\dot c}\,\delta_{\dot\alpha}{}^{\dot c}\,(\theta\theta)\,\theta^\dagger_{\dot e}\,\partial_\nu\lambda^{\dagger\dot e} + 2i\,\sigma^\nu_{b\dot c}\,\theta^{\dagger\dot c}(\theta\theta)\,\epsilon_{\dot e\dot\alpha}\,\partial_\nu\lambda^{\dagger\dot e}$$

$$= -2i\,\sigma^\nu_{b\dot\alpha}\,(\theta\theta)\,\theta^\dagger_{\dot e}\,\partial_\nu\lambda^{\dagger\dot e} + 2i\,\sigma^\nu_{b\dot c}\,\theta^{\dagger\dot c}(\theta\theta)\,\epsilon_{\dot e\dot\alpha}\,\partial_\nu\lambda^{\dagger\dot e}$$

then $\frac{\partial}{\partial\theta^\dagger_{\dot\alpha}}[\text{this}] = -2i\,\sigma^\nu_{b\dot\alpha}\,(\theta\theta)\,\delta^{\dot\alpha}{}_{\dot e}\,\partial_\nu\lambda^{\dagger\dot e} + 2i\,\underbrace{\epsilon^{\dot c\dot\alpha}\,(\theta\theta)\,\epsilon_{\dot e\dot\alpha}}_{-\delta^{\dot c}{}_{\dot e}}\,\partial_\nu\lambda^{\dagger\dot e}$

$$= -2i\,\sigma^\nu_{b\dot\alpha}(\theta\theta)\,\partial_\nu\lambda^{\dagger\dot\alpha} - 2i\,\sigma^\nu_{b\dot c}\,(\theta\theta)\,\partial_\nu\lambda^{\dagger\dot c}$$

$$= -4i\,\sigma^\nu_{b\dot\alpha}\,(\theta\theta)\,\partial_\nu\lambda^{\dagger\dot\alpha}$$

$$= -4i\,(\theta\theta)\,(\sigma^\nu\partial_\nu\lambda^\dagger)_\alpha$$

$$(D^\dagger D^\dagger)D_\alpha V_{W,Z} = -4\lambda_\alpha - 4\theta_\alpha\left[D + i\partial_\mu A^\mu\right] + 4i(\sigma^\nu\bar\sigma^\mu\theta)_\alpha\,\partial_\nu A_\mu - 4i(\theta\theta)(\sigma^\mu\partial_\mu\lambda^\dagger)_\alpha$$

or

$$W_\alpha = \frac{-1}{4}(\uparrow) = \lambda_\alpha + \theta_\alpha\left[D + i\partial_\mu A^\mu\right] - i(\sigma^\nu\bar\sigma^\mu\theta)_\alpha\,\partial_\nu A_\mu + i(\theta\theta)(\sigma^\mu\partial_\mu\lambda^\dagger)_\alpha$$

note: (next pg)

Can make W_α manifestly gauge invariant by writing

$$i\theta_\alpha \partial_\mu A^\mu - i(\sigma^\nu \bar{\sigma}^\mu \theta)_\alpha \partial_\nu A_\mu$$

$$= i\theta_\alpha \partial_\mu A^\mu - \frac{i}{2}\left(\left([\sigma^\nu, \bar{\sigma}^\mu] + \underbrace{\{\sigma^\nu, \bar{\sigma}^\mu\}}_{2g^{\mu\nu}}\right)\theta\right)_\alpha \partial_\nu A_\mu$$

$$= \cancel{i\theta_\alpha \partial_\mu A^\mu} - \frac{i}{2}\left([\sigma^\nu, \bar{\sigma}^\mu]\theta\right)_\alpha \partial_\nu A_\mu - \cancel{i\theta_\alpha \partial_\mu A^\mu} \qquad \sigma^{\mu\nu} = \frac{i}{4}[\sigma^\mu, \bar{\sigma}^\nu] = S_L^{\mu\nu}$$

(generator of boosts and rotations)

$$= -2(\sigma^{\nu\mu}\theta)_\alpha \partial_\nu A_\mu$$

↑ antisymmetrize

$$= -(\sigma^{\nu\mu}\theta)_\alpha F_{\mu\nu}$$

$$= -(\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu}$$

so that

$$\boxed{\begin{aligned} W_\alpha(y,\theta,\theta^\dagger) &= \lambda_\alpha(y) + \theta_\alpha D(y) - (\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu}(y) + i(\theta\theta)(\sigma^\mu \partial_\mu \lambda^\dagger(y))_\alpha \\ W^\alpha(y,\theta,\theta^\dagger) &= \lambda^\alpha(y) + \theta^\alpha D(y) + (\theta\sigma^{\mu\nu})^\alpha F_{\mu\nu}(y) - i(\theta\theta)(\partial_\mu \lambda^\dagger(y)\bar{\sigma}^\mu)^\alpha \end{aligned}}$$

use (Martin 2.78)

Supersymmetric Field Strength spinor.

— Because W_α is gauge invariant, this form is true for any gauge — not just Wess-Zumino gauge

(this would imply all auxillary fields would cancel if carried through calculation).