

Construct F-term out of bilinear:

- only need $\theta\theta$ terms. (6 terms)

$$W^\alpha W_\alpha = \dots + i(\theta\theta) \lambda^\alpha (\sigma^\mu \partial_\mu \lambda^\dagger)_\alpha + \theta^\alpha \theta_\alpha D^2 - \theta^\alpha \sigma^{\mu\nu} \theta_\alpha F_{\mu\nu} D + (\theta\sigma^{\mu\nu})^\alpha \theta_\alpha F_{\mu\nu} D - (\theta\sigma^{\mu\nu} \sigma^{\rho\sigma} \theta) F_{\mu\nu} F_{\rho\sigma} - i(\theta\theta) (\partial_\mu \lambda^\dagger \sigma^\mu \lambda)$$

$$\begin{aligned} & \left\{ \begin{aligned} & -\frac{1}{2}(\theta\theta) \text{Tr}[\sigma^{\mu\nu} \sigma^{\rho\sigma}] && \text{Martin's SUSY (4.1.12)} \\ & = -\frac{1}{2}(\theta\theta) \frac{1}{2}(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} - i\epsilon^{\mu\nu\rho\sigma}) && \text{Martin (2.90)} \\ & \text{Contract with } F_{\mu\nu} F_{\rho\sigma} \\ & = -\frac{1}{4}(\theta\theta) (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} - i\epsilon^{\mu\nu\rho\sigma}) F_{\mu\nu} F_{\rho\sigma} \\ & = -\frac{1}{4}(\theta\theta) (F_{\mu\nu} F^{\mu\nu} - F_{\mu\nu} F^{\overline{\mu\nu}} - i\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}) \\ & = (\theta\theta) \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \frac{i}{4} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right] \end{aligned} \right. \end{aligned}$$

$$W^\alpha W_\alpha = \dots (\theta\theta) \left[i\lambda^\alpha (\sigma^\mu \partial_\mu \lambda^\dagger)_\alpha - i(\partial_\mu \lambda^\dagger \sigma^\mu \lambda) + D^2 - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \frac{i}{4} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right]$$

Terms displayed are same in x^μ and y^μ coordinates.

↙ canonical normalization

$$S = \dots \int d^4x \frac{1}{2} [W^\alpha W_\alpha]_F$$

$$= \dots \int d^4x \left[\frac{1}{2} i\lambda \sigma^\mu \partial_\mu \lambda^\dagger - \frac{1}{2} i(\partial_\mu \lambda^\dagger) \sigma^\mu \lambda + \frac{1}{2} D^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{8} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right]$$

\uparrow IBP: \ominus \uparrow IBP $\rightarrow \ominus$
 Flip: \ominus

not real, but total derivative
 \rightarrow for U(1) theory, no surface term (drop)

$$= \dots \int d^4x \left[\lambda^\dagger i\sigma^\mu \partial_\mu \lambda + \frac{1}{2} D^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$