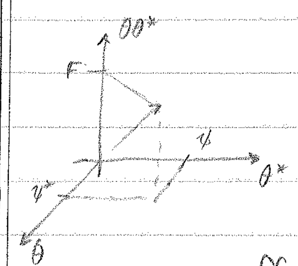
Superfield: $X(t,\theta,\theta^*)$ to be understood as a series in θ & θ^*:

$$X(t,\theta,\theta^*) = x(t) + \sqrt{\frac{\hbar}{m}}\,\theta\,\psi^*(t) + \sqrt{\frac{\hbar}{m}}\,\psi(t)\,\theta^* + \frac{\hbar}{m}\,\theta\theta^* F(t)$$



coefficient functions

$x(t)$, $F(t)$ = even (bosonic) commuting

$\psi(t)$, $\psi^*(t)$ = odd (fermionic) anticommuting

— coefficient functions may be viewed as the "components" of the superfield.

Just as $T_{ij} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix}$ are i,j components of T,

$x(t)$, $\psi(t)$, $\psi^*(t)$, $F(t)$ are four components of X.

We want to build a SUSY invariant Lagrangian out of $X(t,\theta,\theta^*)$
But, for this, we need to find SUSY invariants

How to do this?

Determine irreducible representations — particular combinations will give the relevant invariants.

A superfield $X(t,\theta,\theta^*)$ is usually highly reducible, just as a tensor is highly reducible under the rotation group $SO(3)$.

$$T_{ij} = T\,\delta_{ij} + A_{ij} + S_{ij}$$

Differential forms for $\hat{Q}$ and $\hat{Q}^\dagger$

Recall $H \equiv i\hbar \frac{\partial}{\partial t}$ (H generates time translations)

Match the differential form of Q and $Q^\dagger$ with how it is expected to act on a function of super-coordinates.

$$e^{it'H/\hbar + \alpha Q + \alpha^* Q^\dagger} X(t,\theta,\theta^*)$$

$$= \left(1 + \frac{it'}{\hbar} H + \alpha Q + \alpha^* Q^\dagger + \dots\right) X(t,\theta,\theta^*)$$

$$= X(t,\theta,\theta^*) + \frac{it'}{\hbar} HX + \alpha Q X + \alpha^* Q^\dagger X + \dots \qquad (*)$$

what's supposed to happen under an active transformation:

$$X(t,\theta,\theta^*) \longrightarrow X(t - t' - i\hbar\alpha\theta^* - i\hbar\alpha^*\theta,\ \theta - \alpha,\ \theta^* - \alpha^*)$$

$$\simeq X(t,\theta,\theta^*) + (-t' - i\hbar\alpha\theta^* - i\hbar\alpha^*\theta)\frac{\partial X}{\partial t} - \alpha\frac{\partial X}{\partial \theta} - \alpha^*\frac{\partial X}{\partial \theta^*} + \dots$$

$$= X(t,\theta,\theta^*) - t'\frac{\partial X}{\partial t} - \alpha\left(i\hbar\theta^*\frac{\partial}{\partial t} + \frac{\partial}{\partial\theta}\right)X - \alpha^*\left(i\hbar\theta\frac{\partial}{\partial t} + \frac{\partial}{\partial\theta^*}\right)X \qquad (**)$$

Match (*) with (**)

$$\frac{it'}{\hbar} HX = -t'\frac{\partial X}{\partial t}$$

$$\alpha Q X = -\alpha\left(i\hbar\theta^*\frac{\partial}{\partial t} + \frac{\partial}{\partial\theta}\right)X$$

$$\alpha^* Q^\dagger X = -\alpha^*\left(i\hbar\theta\frac{\partial}{\partial t} + \frac{\partial}{\partial\theta^*}\right)X$$

$$\Rightarrow \boxed{\begin{aligned} H &\equiv i\hbar\frac{\partial}{\partial t} \\ Q &\equiv \frac{-\partial}{\partial\theta} - i\hbar\theta^*\frac{\partial}{\partial t} \\ Q^\dagger &= \frac{-\partial}{\partial\theta^*} - i\hbar\theta\frac{\partial}{\partial t} \end{aligned}}$$

active transformations implemented by $e^{it\hat{H}/\hbar + \alpha Q + \alpha^* Q^\dagger}$

passive " " " $e^{-it\hat{H}/\hbar - \alpha Q - \alpha^* Q^\dagger}$

— Check that differential form for Q and $Q^\dagger$ satisfies SUSY algebra:

$$\{Q, Q^\dagger\} F = \left\{ \frac{-\partial}{\partial\theta} - i\hbar\theta^* \frac{\partial}{\partial t},\ \frac{-\partial}{\partial\theta^*} - i\hbar\theta\frac{\partial}{\partial t} \right\} F$$

test superfunction

$$= \left\{ \frac{-\partial}{\partial\theta}, \frac{-\partial}{\partial\theta^*} \right\} F + i\hbar \overbrace{\left\{ \frac{\partial}{\partial\theta}, \theta \right\}}^{1} \frac{\partial}{\partial t} F + i\hbar \overbrace{\left\{ \theta^*, \frac{\partial}{\partial\theta^*} \right\}}^{1} \frac{\partial}{\partial t} F + (i\hbar)^2 \{\theta^*, \theta\} \frac{\partial^2}{\partial t^2} F$$

$$= 2i\hbar \frac{\partial}{\partial t} F = (2H)\, F \quad ✓$$

Notice that if there were a relative sign between the cross terms in the second line, the anticommutator would vanish.

— can be arranged by flipping sign in middle term of Q or $Q^\dagger$, and defining two new differential operators

$$iD = \frac{-\partial}{\partial\theta} + i\hbar\,\theta^* \frac{\partial}{\partial t}$$

$$-iD^\dagger = \frac{-\partial}{\partial\theta^*} + i\hbar\theta \frac{\partial}{\partial t}$$

SUSY covariant derivative

then $\{Q, -iD^\dagger\} = \{iD, Q^\dagger\} = 0$

and of course $\{Q, iD\} = \{-iD^\dagger, Q^\dagger\} = 0$

— that the anticommutor vanishes will have important consequences for superfield transformations.