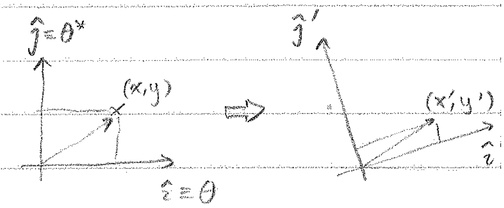
Transformation law for component fields

$$X(t,\theta,\theta^*) = x(t) + \sqrt{\frac{\hbar}{m}}\,\theta\,\psi^*(t) + \sqrt{\frac{\hbar}{m}}\,\psi(t)\,\theta^* + \frac{\hbar}{m}\,\theta\theta^* F(t)$$

Q: How do the components of the superfield transform under supersymmetry transformations?

A: The SUSY transformation of superspace (active)

$$\begin{pmatrix} t \\ \theta \\ \theta^* \end{pmatrix} \longrightarrow \begin{pmatrix} t + i\hbar(\alpha\theta^* + \alpha^*\theta) \\ \theta + \alpha \\ \theta^* + \alpha^* \end{pmatrix}$$



can be viewed as a change of basis, telling us how the components mix into each other

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x\cos\theta + y\sin\theta \\ -x\sin\theta + y\cos\theta \end{pmatrix}$$

Perform a passive SUSY transformation on $X(t,\theta,\theta^*)$ to find out how components x, ψ, ψ^* & F mix under active transformations.

$$X \longrightarrow X + \delta X \qquad Q = \frac{-\partial}{\partial\theta} - i\hbar\theta^*\frac{\partial}{\partial t}$$

$$Q^\dagger = \frac{-\partial}{\partial\theta^*} - i\hbar\theta\frac{\partial}{\partial t}$$

passive transformation

$$\delta X = (-\alpha Q - \alpha^* Q^\dagger)X$$

$$= \alpha\left(i\hbar\theta^*\frac{\partial}{\partial t} + \frac{\partial}{\partial\theta}\right)X + \alpha^*\left(i\hbar\theta\frac{\partial}{\partial t} + \frac{\partial}{\partial\theta^*}\right)X$$

$$= \alpha\, i\hbar\,\theta^*\left(\frac{\partial x}{\partial t} + \sqrt{\frac{\hbar}{m}}\,\theta\,\frac{\partial\psi^*}{\partial t} + 0\right) + \alpha\left(+\sqrt{\frac{\hbar}{m}}\,\psi^* + \frac{\hbar}{m}\theta^* F\right)$$

$$+ \alpha^* i\hbar\,\theta\left(\frac{\partial x}{\partial t} + \sqrt{\frac{\hbar}{m}}\,\frac{\partial\psi}{\partial t}\,\theta^*\right) + \alpha^*\left(-\sqrt{\frac{\hbar}{m}}\,\psi - \frac{\hbar}{m}\theta F\right)$$

expand out all terms ...

$$= \alpha i\hbar\,\theta^* \frac{\partial x}{\partial t} + \alpha i\hbar\,\theta^* \sqrt{\frac{\hbar}{m}}\frac{\partial \psi^*}{\partial t}\,\theta + \alpha\sqrt{\frac{\hbar}{m}}\,\psi^* + \frac{\hbar}{m}\,\alpha\,\theta^* F$$

$$+ \alpha^* i\hbar\,\theta \frac{\partial x}{\partial t} + \alpha^* i\hbar\,\theta \sqrt{\frac{\hbar}{m}}\frac{\partial \psi}{\partial t}\,\theta^* - \alpha^*\sqrt{\frac{\hbar}{m}}\,\psi - \alpha^*\frac{\hbar}{m}\,\theta F$$

- organize terms by powers of θ & θ^*, so that it looks like the expansion of X.

$$= \sqrt{\frac{\hbar}{m}}\left(\alpha\psi^* - \alpha^*\psi\right) + \sqrt{\frac{\hbar}{m}}\,\theta\left[\sqrt{\frac{\hbar}{m}}\,\alpha^*\left(-im\frac{\partial x}{\partial t} + F\right)\right]$$

$$+ \sqrt{\frac{\hbar}{m}}\left[\sqrt{\frac{\hbar}{m}}\,\alpha\left(im\frac{\partial x}{\partial t} + F\right)\right]\theta^* + \frac{\hbar}{m}\,\theta\theta^*\sqrt{\hbar m}\left(i\alpha\frac{\partial\psi^*}{\partial t} - i\alpha^*\frac{\partial\psi}{\partial t}\right)$$

match with $X \sim x + \theta\psi^* + \psi\theta^* + \theta\theta^* F$

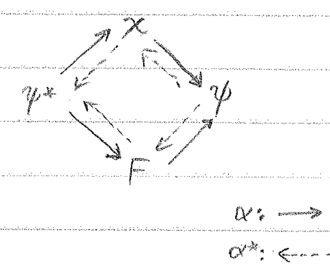
<u>result:</u>

$$x \longrightarrow x + \sqrt{\frac{\hbar}{m}}\left(\alpha\psi^* - \alpha^*\psi\right)$$

$$\psi \longrightarrow \psi + \sqrt{\frac{\hbar}{m}}\,\alpha\left(im\frac{\partial x}{\partial t} + F\right)$$

$$\psi^* \longrightarrow \psi^* + \sqrt{\frac{\hbar}{m}}\,\alpha^*\left(-im\frac{\partial x}{\partial t} + F\right)$$

$$F \longrightarrow F + \sqrt{\hbar m}\left(i\alpha\frac{\partial\psi^*}{\partial t} - i\alpha^*\frac{\partial\psi}{\partial t}\right)$$



This is the defining transformation law for a superfield.
(a function $N(t,\theta,\theta^*)$ whose components do not transform in the way above is <u>not</u> a superfield)

<u>Important to note:</u> F transforms into a total derivative of time

$$F \sim F + \frac{\partial}{\partial t}(\cdot)$$

→ this will be a useful property to keep in mind when building SUSY Lagrangians.