Properties:

① Sum of two superfields is also a superfield
② Product of two superfields is also a superfield

Meaning of above statements:

Let $X(t,\theta,\theta^*) = x + \sqrt{\frac{\hbar}{m}}\,\theta\psi^* + \sqrt{\frac{\hbar}{m}}\,\psi\theta^* + \frac{\hbar}{m}\theta\theta^* F$

$Y(t,\theta,\theta^*) = y + \sqrt{\frac{\hbar}{m}}\,\theta\xi^* + \sqrt{\frac{\hbar}{m}}\,\xi\theta^* + \frac{\hbar}{m}\theta\theta^* G$

be two superfields (which is a statement about how $\overset{x}{y}, \overset{\psi}{\xi}, \overset{\psi^*}{\xi^*}, \overset{F}{G}$ transform),

namely $x \to x + \sqrt{\frac{\hbar}{m}}(\alpha\psi^* - \alpha^*\psi)$

$y \to y + \sqrt{\frac{\hbar}{m}}(\alpha\xi^* - \alpha^*\xi)$

Check property #2:

$$X(t,\theta,\theta^*)\,Y(t,\theta,\theta^*)$$

$$= \left(x + \sqrt{\tfrac{\hbar}{m}}\,\theta\psi^* + \sqrt{\tfrac{\hbar}{m}}\,\psi\theta^* + \tfrac{\hbar}{m}\theta\theta^* F\right)\left(y + \sqrt{\tfrac{\hbar}{m}}\,\theta\xi^* + \sqrt{\tfrac{\hbar}{m}}\,\xi\theta^* + \tfrac{\hbar}{m}\theta\theta^* G\right)$$

$$= xy + \sqrt{\tfrac{\hbar}{m}}\,x\theta\xi^* + \sqrt{\tfrac{\hbar}{m}}\,x\xi\theta^* + \tfrac{\hbar}{m}\theta\theta^* xG$$

$$+ \sqrt{\tfrac{\hbar}{m}}\,\theta\psi^* y + \tfrac{\hbar}{m}\theta\psi^*\xi\theta^* + \sqrt{\tfrac{\hbar}{m}}\,\psi\theta^* y + \tfrac{\hbar}{m}\psi\theta^*\theta\xi^* + \tfrac{\hbar}{m}\theta\theta^* F y$$

$$= xy + \sqrt{\tfrac{\hbar}{m}}\,\theta(x\xi^* + y\psi^*) + \sqrt{\tfrac{\hbar}{m}}(x\xi + y\psi)\theta^*$$

$$+ \tfrac{\hbar}{m}\theta\theta^*(xG + yF + \psi^*\xi + \xi^*\psi)$$

Terminology: coefficient of "1" of superfield is lowest/bottom component
" " $\theta\theta^*$ of superfield is highest/top component.

Each of these four coefficients must transform according to the defining property of a superfield.

e.g. the bottom component must transform

$$(xy) \to (xy) + \sqrt{\tfrac{\hbar}{m}}\left[\alpha(x\xi^* + y\psi^*) - \alpha^*(x\xi + y\psi)\right]$$

check using transformation law of x & y:

$$(xy) \longrightarrow \left(x + \sqrt{\tfrac{\hbar}{m}}\,(\alpha \psi^* - \alpha^* \psi)\right)\left(y + \sqrt{\tfrac{\hbar}{m}}\,(\alpha \xi^* - \alpha^* \xi)\right)$$

$$= \overbrace{xy}^{\text{square term}} + \overbrace{\sqrt{\tfrac{\hbar}{m}}\, x(\alpha \xi^* - \alpha^* \xi) + \sqrt{\tfrac{\hbar}{m}}\, y(\alpha \psi^* - \alpha^* \psi)}^{\text{cross terms}} + \overbrace{O(\alpha)^2}^{\text{higher order}}$$

$$= xy + \sqrt{\tfrac{\hbar}{m}}\left[\alpha(x\xi^* + y\psi^*) - \alpha^*(x\xi + y\psi)\right]$$

match !

easy to check other components match.

Reason this works is infinitessimal SUSY varations obey Leibniz rule:

$$xy \longrightarrow xy + \delta(xy)$$
$$= xy + (\delta x)y + x(\delta y)$$

—thanks to the fact that SUSY is linearized on SUSY space.

example when this does not work:

$$x \longrightarrow x + \alpha x^2 \qquad \text{(non-linear)}$$

$$(x^2) \longrightarrow (x^2) + \alpha (x^2)^2$$

$$xx \longrightarrow (x + \alpha x^2)(x + \alpha x^2) = x^2 + 2\alpha x^3 + O(\alpha^2)$$