<u>Covariant derivative</u>

Recall, $\quad D = i\dfrac{\partial}{\partial\theta} + \hbar\theta^*\dfrac{\partial}{\partial t}$

$\qquad\quad D^\dagger = -i\dfrac{\partial}{\partial\theta^*} - \hbar\theta\dfrac{\partial}{\partial t}$

③ The action of a covariant derivative on a superfield is also a superfield. — hence the "covariant" name.

$$DX = \left(i\frac{\partial}{\partial\theta} + \hbar\theta^*\frac{\partial}{\partial t}\right)\left(x + \sqrt{\frac{\hbar}{m}}\theta\psi^* + \sqrt{\frac{\hbar}{m}}\psi\theta^* + \frac{\hbar}{m}\theta\theta^* F\right)$$

$$= \left(i\sqrt{\frac{\hbar}{m}}\psi^* + \frac{\hbar}{m}i\theta^* F\right) + \hbar\theta^*\left(\frac{\partial x}{\partial t} + \sqrt{\frac{\hbar}{m}}\theta\frac{\partial\psi^*}{\partial t}\right)$$

$$= \underline{i\sqrt{\frac{\hbar}{m}}\psi^* + \sqrt{\frac{\hbar}{m}}\left[\sqrt{\frac{\hbar}{m}}\left(m\frac{\partial x}{\partial t} + iF\right)\theta^*\right] - \frac{\hbar}{m}\theta\theta^*\left(\sqrt{\hbar m}\frac{\partial\psi^*}{\partial t}\right)}$$

similarly,

$$D^\dagger X = \left(-i\frac{\partial}{\partial\theta^*} - \hbar\theta\frac{\partial}{\partial t}\right)\left(x + \sqrt{\frac{\hbar}{m}}\theta\psi^* + \sqrt{\frac{\hbar}{m}}\psi\theta^* + \frac{\hbar}{m}\theta\theta^* F\right)$$

$$= i\sqrt{\frac{\hbar}{m}}\psi + i\frac{\hbar}{m}\theta F - \hbar\theta\frac{\partial x}{\partial t} - \hbar\sqrt{\frac{\hbar}{m}}\theta\frac{\partial\psi}{\partial t}\theta^*$$

$$= \underline{i\sqrt{\frac{\hbar}{m}}\psi + \sqrt{\frac{\hbar}{m}}\theta\left[\sqrt{\frac{\hbar}{m}}\left(-m\frac{\partial x}{\partial t} + iF\right)\right] + \frac{\hbar}{m}\theta\theta^*\left(\sqrt{\hbar m}\frac{\partial\psi}{\partial t}\right)}$$

c.f.: $x \to x + \sqrt{\frac{\hbar}{m}}(\alpha\psi^* - \alpha^*\psi)$

Check: $\left(i\sqrt{\frac{\hbar}{m}}\psi\right) \to \left(i\sqrt{\frac{\hbar}{m}}\psi\right) + \sqrt{\frac{\hbar}{m}}\alpha\left[\sqrt{\frac{\hbar}{m}}\left(-m\frac{\partial x}{\partial t} + iF\right)\right] - 0$

factor out $i\sqrt{\frac{\hbar}{m}}$

$$= i\sqrt{\frac{\hbar}{m}}\Big(\underbrace{\psi + \sqrt{\frac{\hbar}{m}}\alpha\left(im\frac{\partial x}{\partial t} + F\right)}_{\text{just as how } \psi \text{ transforms.}}\Big) \checkmark$$

as expected because $DX \to DX + D\delta X$

$\qquad = DX + D(\alpha Q + \alpha^* Q^\dagger)X$

But $\;[D, \alpha Q] = [D, \alpha Q^\dagger] = 0$

Then, $DX \longrightarrow DX + (\alpha Q + \alpha^* Q^\dagger) DX$

$$= (DX) + \delta(DX)$$ hence, covariant.

Notice, however, that the action of "D" converts the superfield from Grassmann <u>even</u> to Grassmann <u>odd</u>.