Superanalytic functions

Reminder in complex analysis:

If I have an analytic function $f: \mathbb{R} \to \mathbb{C}$ at point $x=0$,

— meaning I have a Taylor series defined at $x=0$:

$$f(x) = f(0) + x f'(0) + \frac{1}{2} x^2 f''(0) + \dots$$

coefficients may be complex-valued,

then the extension to complex arguments is straightforward:

Wherever there is an x, replace it by $z \in \mathbb{C}$. (accessing $f(0)$, $f'(0)$, ... defines derivative w.r.t complex numbers)

For extensions to supernumbers, best to consider continuations to Grassman-odd (anticommuting) and Grassmann-even (commuting) numbers separately.

① Continuing $\mathbb{C} \to \Lambda_\infty$ to $\Lambda_N^a \to \Lambda_\infty$:

Start with Taylor series about $z=0$ (Grassmann-odd numbers have no body)

$f: \mathbb{C} \to \Lambda_\infty$ $\quad f(z) = f(0) + z f'(0) + \frac{1}{2} z^2 f''(0) + \dots$

coefficients are supernumbers Λ_∞.

replace z with $\theta \in \Lambda_N^a$

$$f(\theta) = f(0) + \theta f'(0) \quad \text{(series terminates)}$$

Ability to access $f(0)$ straightforward (set argument to zero)

to access $f'(0)$, define derivative w.r.t. Grassmann odd variables:

Left derivative: $\quad \frac{\overrightarrow{\partial}}{\partial\theta} f(\theta) = \frac{\partial}{\partial\theta}\left(f(0) + \theta f'(0)\right) = f'(0)$

right derivative: $\quad f(\theta)\frac{\overleftarrow{\partial}}{\partial\theta} = \left(f(0) + \theta f'(0)\right)\frac{\overleftarrow{\partial}}{\partial\theta} = \left(\theta f'(0)\right)\frac{\overleftarrow{\partial}}{\partial\theta}$

(non-trivial)

② Continuation $\mathbb{C} \to \Lambda_\infty$ to $\Lambda_N^c \to \Lambda_\infty$:

– more straightforward.

Start, again with Taylor series about $z = c_B$ ← would-be body of element in Λ_N^c

$f: \mathbb{C} \to \Lambda_\infty \quad f(z) = f(0) + z f'(0) + \frac{1}{2} z^2 f''(0) + \ldots$

replace z with $c \in \Lambda_N^c$.

$c_S = c - c_B$

$$f(c) = f(c_B) + c_S f'(c_B) + \frac{1}{2} c_S^2 f''(c_B) + \ldots$$

(series continues indefinitely)

Analysis of superanalytic function on Grassmann-even supernumbers proceeds in an analogous manner to complex analysis:

<u>results</u>:

– $$f(c) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(c_B)\, c_S^n$$

– Consider a closed curve γ in vector space of Λ_N^c, then

$$\oint_\gamma f(c)\, dc = 0 \qquad \text{if } \gamma \equiv \text{contractible}.$$

$$\oint_\gamma f(c)\, dc = 2\pi i \sum_i \text{Res}\, f(c_i)$$

coefficient of $1/c$ in "Laurent expansion"

(c has a body),