Brief Review of

Symplectic Structure and canonical quantization

I can write action in canonical form by inverting the Legendre transform.

Hamilton's action : $S[x,p] = \int dt \left(\underbrace{p\dot{x}} - H(x,p) \right)$

Symplectic term
fixes Poisson bracket of phase space.

recall that Euler-Lagrange equations of motion derived from this action gives Hamilton's equations.

Symmetric form:
- obtained by integrating by parts

$$S[x,p] = \int dt \left[\frac{1}{2} (p\dot{x} - x\dot{p}) - H(x,p) \right]$$

Assemble phase space coordinates into a vector: $z = \begin{pmatrix} x \\ p \end{pmatrix}$
and write

$$S[z] = \int dt \left[\frac{1}{2} z^a (\Omega^{-1})_{ab} \dot{z}^b - H(z) \right]$$

$$(\Omega^{-1})_{ab} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \Rightarrow \Omega^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Poisson bracket of two functions on phase space:

$$\{F, G\}_{PB} = \frac{\partial F}{\partial z^a} \Omega^{ab} \frac{\partial G}{\partial z^b}$$

satisfies antisymmetry
Leibniz rule
Jacobi identity

Canonical quantization obtained by representing algebra on Hilbert space:

$$[\hat{z}^a, \hat{z}^b] = i\hbar\, \Omega^{ab}$$

$$[F(\hat{z}), G(\hat{z})] = i\hbar \{F, G\}_{P.B} + O(\hbar^2)$$