Canonical Quantization:

$$\{\hat{\chi}_a(t,\vec{x}),\ \hat{\pi}^b(t,\vec{x}')\} = i\,\delta^{(3)}(\vec{x}-\vec{x}')\,\delta_a{}^b \leftarrow$$ equal time ANTI-commutation relation.

$$\text{but}\quad \hat{\pi}^b(t,\vec{x}') = \frac{\partial \mathcal{L}}{\partial \dot{\chi}_b} = i\chi^\dagger_{\dot c}\,\bar{\sigma}^{\dot c a}$$

$$\{\hat{\chi}_a(t,\vec{x}),\ i\chi^\dagger_{\dot c}\,\bar{\sigma}^{\dot c a}\} = i\,\delta^{(3)}(\vec{x}-\vec{x}')\,\delta_a{}^b$$

Proceed to evaluate LHS

$$\text{LHS} = \int\frac{d^3p}{(2\pi)^3}\int\frac{d^3p'}{(2\pi)^3}\frac{1}{2\sqrt{\omega_p\omega_{p'}}}\ \overset{\text{defn of }\hat{\pi}^b}{i}\sum_{s=1,2}\sum_{s'=1,2}$$
$$\Big(\{\hat{a}_{\vec{p},s},\ \hat{a}^\dagger_{\vec{p}',s'}\}\,u_a^{[s]}(\vec{p})\,u^{\dagger[s']}_{\dot c}(\vec{p}')\,\bar{\sigma}^{\dot c b 0}\,e^{-i(\omega_p-\omega_{p'})t+i\vec{p}\cdot\vec{x}-i\vec{p}'\cdot\vec{x}'}$$
$$+\{\hat{a}^\dagger_{\vec{p},s},\ \hat{a}_{\vec{p}',s'}\}\,v_a^{[s]}(\vec{p})\,v^{\dagger[s']}_{\dot c}(\vec{p}')\,\bar{\sigma}^{\dot c b 0}\,e^{+i(\omega_p-\omega_{p'})t-i\vec{p}\cdot\vec{x}+i\vec{p}'\cdot\vec{x}'}\Big)$$

Evaluate anticommutators
$$\{\hat{a}_{\vec{p},s},\ \hat{a}^\dagger_{\vec{p}',s'}\} = (2\pi)^3\,\delta^{(3)}(\vec{p}-\vec{p}')\,\delta_{ss'}$$
$$\{\hat{a}^\dagger_{\vec{p},s},\ \hat{a}_{\vec{p}',s'}\} = +(2\pi)^3\,\delta^{(3)}(\vec{p}-\vec{p}')\,\delta_{ss'}$$

- sum over s', fixing $s'\to s$
- integrate over $\vec{p}'$, fixing $\vec{p}'\to\vec{p}$

$$\text{LHS} = i\int\frac{d^3p}{(2\pi)^3}\frac{1}{2\omega_p}\sum_{s=1,2}\Big(u_a^{[s]}(\vec{p})\,u^{\dagger[s]}_{\dot c}(\vec{p})\,\bar{\sigma}^{\dot c b 0}\,e^{i\vec{p}\cdot(\vec{x}-\vec{x}')}$$
$$+ v_a^{[s]}(\vec{p})\,v^{\dagger[s]}_{\dot c}(\vec{p})\,\bar{\sigma}^{\dot c b 0}\,e^{-i\vec{p}\cdot(\vec{x}-\vec{x}')}\Big)$$

Perform sum over s, using completeness relations:
$$\sum_s u_a^{[s]}(\vec{p})\,u^{\dagger[s]}_{\dot c}(\vec{p}) = p_\mu\sigma^\mu_{a\dot c} = \omega_p\sigma^0_{a\dot c}-\vec{p}\cdot\vec{\sigma}_{a\dot c}$$
$$\sum_s v_a^{[s]}(\vec{p})\,v^{\dagger[s]}_{\dot c}(\vec{p}) = p_\mu\sigma^\mu_{a\dot c} = \omega_p\sigma^0_{a\dot c}-\vec{p}\cdot\vec{\sigma}_{a\dot c}$$

$$\text{LHS} = i\int\frac{d^3p}{(2\pi)^3}\frac{1}{2\omega_p}\Big((\omega_p\sigma^0_{a\dot c}-\vec{p}\cdot\vec{\sigma}_{a\dot c})\,\bar{\sigma}^{\dot c b 0}\,e^{i\vec{p}\cdot(\vec{x}-\vec{x}')}+(\omega_p\sigma^0_{a\dot c}-\vec{p}\cdot\vec{\sigma}_{a\dot c})\,\bar{\sigma}^{\dot c b 0}\,e^{-i\vec{p}\cdot(\vec{x}-\vec{x}')}$$

In second term, change integration variables $\vec{p} \longrightarrow -\vec{p}$:

$$LHS = i\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{2\omega_p}\left(\left(\omega_p \sigma^0_{a\dot{c}} - \cancel{\vec{p}\cdot\vec{\sigma}_{a\dot{c}}}\right) \bar{\sigma}^{\dot{c}b0}\, e^{i\vec{p}\cdot(\vec{x}-\vec{x}')} + \left(\omega_p \sigma^0_{a\dot{c}} + \cancel{\vec{p}\cdot\vec{\sigma}_{a\dot{c}}}\right) \bar{\sigma}^{\dot{c}b0}\, e^{+i\vec{p}\cdot(\vec{x}-\vec{x}')} \right)$$

add.

$$= i\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{2\omega_p}\, 2\times \omega_p\, \underbrace{\sigma^0_{a\dot{c}}\,\bar{\sigma}^{\dot{c}b0}}_{\delta_a{}^b}\, e^{i\vec{p}\cdot(\vec{x}-\vec{x}')}$$

$$= i\int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\cdot(\vec{x}-\vec{x}')}\,\delta_a{}^b$$

$$= i\,\delta^{(3)}(\vec{x}-\vec{x}')\,\delta_a{}^b \;=\; RHS \;\checkmark$$