Dreiner-Haber-Martin stacking and splitting formulas

Let $\psi = \begin{pmatrix} \chi \\ \xi^\dagger \end{pmatrix}$ then $\overline{\psi} = \begin{pmatrix} \xi & \chi^\dagger \end{pmatrix}$

$\psi^c = \begin{pmatrix} \xi \\ \chi^\dagger \end{pmatrix}$ $\qquad \overline{\psi^c} = \begin{pmatrix} \chi & \xi^\dagger \end{pmatrix}$

Dirac side:
$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

Weyl side:
$\sigma^{\mu\nu} = \frac{i}{4}[\sigma^\mu, \bar\sigma^\nu] = S_L^{\mu\nu}$
$\bar\sigma^{\mu\nu} = \frac{i}{4}[\bar\sigma^\mu, \sigma^\nu] = S_R^{\mu\nu}$

Stacking (2-component $\Rightarrow$ 4 component):

atypical bilinears: (obtained by charge conj.)

$$\begin{cases} \xi^i \chi_j = \overline{\Psi}^i \hat{P}_L \psi_j \\ \chi^{\dagger i} \xi^\dagger_j = \overline{\Psi}^i \hat{P}_R \psi_j \end{cases} \qquad \begin{aligned} \chi^i \chi_j &= \overline{\Psi}^i \hat{P}_L \psi_j^c \\ \chi^{\dagger i} \chi^\dagger_j &= \overline{\Psi}^i \hat{P}_R \psi_j^c \end{aligned}$$

$$\begin{cases} \chi^{\dagger i} \bar\sigma^\mu \chi_j = \overline{\Psi}^i \gamma^\mu \hat{P}_L \psi_j \\ \xi^i \sigma^\mu \xi^\dagger_j = \overline{\Psi}^i \gamma^\mu \hat{P}_R \psi_j \end{cases} \qquad \begin{aligned} \chi^{\dagger i} \bar\sigma^\mu \xi_j &= \overline{\Psi}^i \gamma^\mu \hat{P}_L \psi_j^c \\ \xi^i \sigma^\mu \chi^\dagger_j &= \overline{\Psi}^i \gamma^\mu \hat{P}_R \psi_j^c \end{aligned}$$

$$\begin{cases} 2\xi^i \sigma^{\mu\nu} \chi_j = \overline{\Psi}^i \sigma^{\mu\nu} \hat{P}_L \psi_j \\ 2\chi^{\dagger i} \bar\sigma^{\mu\nu} \xi^\dagger_j = \overline{\Psi}^i \sigma^{\mu\nu} \hat{P}_R \psi_j \end{cases} \qquad \begin{aligned} 2\xi^i \sigma^{\mu\nu} \xi_j &= \overline{\Psi}^i \sigma^{\mu\nu} \hat{P}_L \psi_j^c \\ 2\chi^\dagger \bar\sigma^{\mu\nu} \chi^\dagger_j &= \overline{\Psi}^i \sigma^{\mu\nu} \hat{P}_R \psi_j^c \end{aligned}$$

Splitting (2-component $\Leftarrow$ 4-component):

atypical bilinears

(S) $\overline{\Psi}^i \psi_j = \xi^i \chi_j + \chi^{\dagger i} \xi^\dagger_j$ $\qquad \overline{\Psi}^i \psi_j^c = \xi^i \xi_j + \chi^{\dagger i} \chi^\dagger_j$

(P) $\overline{\Psi}^i \gamma_5 \psi_j = -\xi^i \chi_j + \chi^{\dagger i} \xi^\dagger_j$ $\qquad \overline{\Psi}^i \gamma_5 \psi_j^c = -\xi^i \xi_j + \chi^{\dagger i} \chi^\dagger_j$

$\vdots$

replace in left spinor
$\chi \leftrightarrow \xi$

(V) $\overline{\Psi}^i \gamma^\mu \psi_j = \chi^{\dagger i} \bar\sigma^\mu \chi_j + \xi^i \sigma^\mu \xi^\dagger_j$

(A) $\overline{\Psi}^i \gamma^\mu \gamma_5 \psi_j = -\chi^{\dagger i} \bar\sigma^\mu \chi_j + \xi^i \sigma^\mu \xi^\dagger_j$

(T) $\overline{\Psi}^i \sigma^{\mu\nu} \psi_j = 2(\xi^i \sigma^{\mu\nu} \chi_j + \chi^{\dagger i} \bar\sigma^{\mu\nu} \xi^\dagger_j)$

( ) $\overline{\Psi}^i \sigma^{\mu\nu} \gamma_5 \psi_j = 2(-\xi^i \sigma^{\mu\nu} \chi_j + \chi^{\dagger i} \bar\sigma^{\mu\nu} \xi^\dagger_j)$

Equations valid for commuting & anticommuting spinors.

For Majorana, set $\xi^i = \chi_i$ and $\xi^\dagger_i = \chi^{\dagger i}$.

(if $i=j$ for any bilinear with Maj. spinors, vector (V), tensor (T) and pseudotensors ( ) vanish)