Summary of active rotation/boost matrix

$$\theta_{\rho\sigma} M^{\rho\sigma} = \theta_{01} M^{01} + \theta_{02} M^{02} + \theta_{03} M^{03} + \theta_{12} M^{12} + \theta_{13} M^{13} + \theta_{23} M^{23}$$

$$+ \theta_{10} M^{10} + \theta_{20} M^{20} + \theta_{30} M^{30} + \theta_{21} M^{21} + \theta_{31} M^{31} + \theta_{32} M^{32}$$

$$= 2\underbrace{\left(\theta_{01} M^{01} + \theta_{02} M^{02} + \theta_{03} M^{03}\right)}_{\text{Boost}} + 2\underbrace{\left(\theta_{12} M^{12} + \theta_{13} M^{13} + \theta_{23} M^{23}\right)}_{\text{Rotation.}}$$

Parameters		Generators	
$\theta_{01} = \eta_x$	$\theta_{12} = \theta_z$	$M^{01} = K^1$	$M^{12} = J_z$
$\theta_{02} = \eta_y$	$\theta_{13} = -\theta_y$	$M^{02} = K^2$	$M^{13} = -J_y$
$\theta_{03} = \eta_z$	$\theta_{23} = \theta_x$	$M^{03} = K^3$	$M^{23} = J_z$
(Rapidity)	(Angles)		

$$= 2\left(\vec{\eta}\cdot\vec{K} + \vec{\theta}\cdot\vec{J}\right)$$

So that

$$\Lambda(\theta)^{\mu}{}_{\nu} = \left(e^{-\frac{i}{2}\theta_{\rho\sigma} M^{\rho\sigma}}\right)^{\mu}{}_{\nu} = e^{-i(\vec{\eta}\cdot\vec{K} + \vec{\theta}\cdot\vec{J})}$$

is the active rotation/boost matrix.

$$J_x = \left(\begin{array}{c|ccc} 0 & & & \\ \hline & 0 & & \\ & & 0 & -i \\ & & i & 0 \end{array}\right) \qquad J_y = \left(\begin{array}{c|ccc} 0 & & & \\ \hline & 0 & & i \\ & & 0 & \\ & -i & & 0 \end{array}\right) \qquad J_z = \left(\begin{array}{c|ccc} 0 & & & \\ \hline & 0 & -i & \\ & i & 0 & \\ & & & 0 \end{array}\right)$$

$$K_x = \left(\begin{array}{c|ccc} 0 & i & 0 & 0 \\ \hline i & & & \\ 0 & & & \\ 0 & & & \end{array}\right) \qquad K_y = \left(\begin{array}{c|ccc} 0 & 0 & i & 0 \\ \hline 0 & & & \\ i & & & \\ 0 & & & \end{array}\right) \qquad K_z = \left(\begin{array}{c|ccc} 0 & 0 & 0 & i \\ \hline 0 & & & \\ 0 & & & \\ i & & & \end{array}\right)$$

(Active) Boost matrix for vectors (spin-1 objects)

$$\Lambda^\mu{}_\nu(\vec{\eta}) = e^{-i\vec{\eta}\cdot\vec{K}}$$

$$K_1 = \left(\begin{array}{c|ccc} & i & 0 & 0 \\ \hline i & & & \\ 0 & & & \\ 0 & & & \end{array}\right) \qquad K_2 = \left(\begin{array}{c|ccc} & 0 & i & 0 \\ \hline 0 & & & \\ i & & & \\ 0 & & & \end{array}\right) \qquad K_3 = \left(\begin{array}{c|ccc} & 0 & 0 & i \\ \hline 0 & & & \\ 0 & & & \\ i & & & \end{array}\right)$$

$$\Lambda^\mu{}_\nu(\vec{\eta}) = \left(\begin{array}{c|c} \cosh\eta & \hat{\eta}\sinh\eta \\ \hline \hat{\eta}^T\sinh\eta & \delta_{ij} + \hat{\eta}_i\hat{\eta}_j(\cosh\eta - 1) \end{array}\right)$$

In terms of velocity / gamma factor: $\cosh\eta = \gamma$, $\sinh\eta = |\vec{\beta}|\gamma$, $\hat{\eta} = \hat{\beta}$ & $\hat{\beta}|\vec{\beta}| = \vec{\beta}$

$$\Lambda^\mu{}_\nu(\vec{\beta}) = \left(\begin{array}{c|c} \gamma & \vec{\beta}\gamma \\ \hline \vec{\beta}^T\gamma & \delta_{ij} + \hat{\beta}_i\hat{\beta}_j(\gamma - 1) \end{array}\right)$$

$$\left\{ \begin{array}{l} \text{n.b. for } \vec{\eta} = (\eta, 0, 0) \text{ or } \vec{\beta} = (\beta, 0, 0), \\[2ex] \Lambda^\mu{}_\nu(\beta_x) = \left(\begin{array}{c|ccc} \gamma & \beta_x\gamma & & \\ \hline \beta_x\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{array}\right) \end{array} \right.$$

Since $\hat{\beta} = (1, 0, 0)$ (unit-vector), $\hat{\beta}_i\hat{\beta}_j = \delta_{ij}$

Then, in terms of energy, momentum,

$\gamma = E/m$, $\vec{\beta}\gamma = \vec{p}/m$, and
$$\delta_{ij} + \hat{\beta}_i\hat{\beta}_j(\gamma - 1) = \delta_{ij} + \frac{p_i p_j}{|\vec{p}|^2}\left(\frac{E}{m} - 1\right)$$
$$= \delta_{ij} + \frac{p_i p_j}{(E^2 - m^2)}\left(\frac{E - m}{m}\right)$$
$$= \delta_{ij} + \frac{p_i p_j}{m(E+m)}$$

$$\Lambda^\mu{}_\nu(\vec{p}) = \left(\begin{array}{c|c} E/m & \vec{p}/m \\ \hline \vec{p}^T/m & \delta_{ij} + \dfrac{p_i p_j}{m(E+m)} \end{array}\right)$$